

## SYLLABUS

### 1. Information regarding the programme

1.1 Higher education institution	<b>Babeş-Bolyai University Cluj-Napoca</b>
1.2 Faculty	<b>Faculty of Mathematics and Computer Science</b>
1.3 Department	<b>Department of Mathematics</b>
1.4 Field of study	<b>Mathematics</b>
1.5 Study cycle	<b>Master</b>
1.6 Study programme / Qualification	<b>Advanced Mathematics</b>

### 2. Information regarding the discipline

2.1 Name of the discipline (en) (ro)		<b>Integral Equations with Applications</b> <b>Ecuatii integrale cu aplicații</b>					
2.2 Course coordinator		<b>Prof. Sanda Micula, PhD. Habil.</b>					
2.3 Seminar coordinator		<b>Prof. Sanda Micula, PhD. Habil.</b>					
2.4. Year of study	<b>2</b>	2.5 Semester	<b>3</b>	2.6. Type of evaluation	<b>E</b>	2.7 Type of discipline	<b>DS Optional</b>
2.8 Code of the discipline		<b>MME3160</b>					

### 3. Total estimated time (hours/semester of didactic activities)

3.1 Hours per week	<b>3</b>	Of which: 3.2 course	<b>2</b>	3.3 seminar/laboratory	<b>1 sem</b>
3.4 Total hours in the curriculum	<b>42</b>	Of which: 3.5 course	<b>28</b>	3.6 seminar/laboratory	<b>14</b>
Time allotment:					hours
Learning using manual, course support, bibliography, course notes					40
Additional documentation (in libraries, on electronic platforms, field documentation)					30
Preparation for seminars/labs, homework, papers, portfolios and essays					40
Tutorship					14
Evaluations					9
Other activities: .....					
3.7 Total individual study hours					133
3.8 Total hours per semester					175
3.9 Number of ECTS credits					7

### 4. Prerequisites (if necessary)

4.1. curriculum	<ul style="list-style-type: none"> <li>• Mathematical Analysis, Numerical Analysis</li> </ul>
4.2. competencies	<ul style="list-style-type: none"> <li>• Knowledge of basic notions of operator theory</li> <li>• Average programming skills</li> </ul>

### 5. Conditions (if necessary)

5.1. for the course	<ul style="list-style-type: none"> <li>• Classroom with large blackboard and video projector</li> </ul>
5.2. for the seminar /lab activities	<ul style="list-style-type: none"> <li>• Classroom with large blackboard and video projector/computers with Matlab</li> </ul>

## 6. Specific competencies acquired

<b>Professional competencies</b>	<ul style="list-style-type: none"> <li>• Ability to understand and manipulate concepts, results and advanced mathematical theories.</li> <li>• Ability to model and analyze from the mathematical point of view real processes from other sciences, economics, and engineering.</li> <li>• Ability to use the scientific language and to write scientific reports and papers.</li> </ul>
<b>Transversal competencies</b>	<ul style="list-style-type: none"> <li>• Ability to inform themselves, to work independently or in a team in order to realize studies and to solve complex problems.</li> <li>• Ability for continuous self-perfecting and study.</li> <li>• Ability to use advanced and complementary knowledge in order to obtain a PhD in Pure Mathematics and Applied Mathematics.</li> </ul>

## 7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	<ul style="list-style-type: none"> <li>• Acquire knowledge of the general theory of integral equations, with focus on applications.</li> <li>• Gain the ability to apply concepts and results from integral equations theory to specific problems.</li> </ul>
7.2 Specific objective of the discipline	<ul style="list-style-type: none"> <li>• Understand and be able to use main concepts and results from general integral equations theory.</li> <li>• Be able to analyze the solvability of specific integral equations arising in applications.</li> <li>• Understand, use and be able to derive numerical methods for the approximate solution of integral equations arising in applications from various fields.</li> </ul>

## 8. Content

8.1 Course	Teaching methods	Remarks
<b>1. Introduction.</b> Basic concepts. History of integral equations. Classifications and examples.	<ul style="list-style-type: none"> <li>• Interactive exposure</li> <li>• Explanation</li> <li>• Conversation</li> <li>• Description</li> </ul>	
<b>2.</b> Types of integral equations with exact solutions.	<ul style="list-style-type: none"> <li>• Interactive exposure</li> <li>• Explanation</li> <li>• Conversation</li> <li>• Description</li> </ul>	
<b>3.</b> Relationship between initial value/boundary value problems and integral equations.	<ul style="list-style-type: none"> <li>• Interactive exposure</li> <li>• Explanation</li> <li>• Conversation</li> <li>• Description</li> </ul>	
<b>4. Volterra integral equations.</b> The method of successive approximations. Laplace transforms. Adomian decomposition.	<ul style="list-style-type: none"> <li>• Interactive exposure</li> <li>• Explanation</li> <li>• Conversation</li> <li>• Description</li> </ul>	

5. Series solution. Volterra integral equations of the first kind. Integral equations of the convolution type. Abel integral equation.	<ul style="list-style-type: none"> <li>• Interactive exposure</li> <li>• Explanation</li> <li>• Conversation</li> <li>• Description</li> </ul>	
6. Fredholm integral equations. The method of successive approximations, Neumann series. Adomian decomposition. Compact integral operators. Properties. The Fredholm alternative theorem.	<ul style="list-style-type: none"> <li>• Interactive exposure</li> <li>• Explanation</li> <li>• Conversation</li> <li>• Description</li> </ul>	
7. Homogeneous Fredholm equations. Fredholm integral equations of the first kind.	<ul style="list-style-type: none"> <li>• Interactive exposure</li> <li>• Explanation</li> <li>• Conversation</li> <li>• Description</li> </ul>	
8. Numerical methods. Degenerate kernel methods. Taylor series approximation. Interpolatory degenerate kernel approximation.	<ul style="list-style-type: none"> <li>• Interactive exposure</li> <li>• Explanation</li> <li>• Conversation</li> <li>• Description</li> </ul>	
9. Projection methods, collocation and Galerkin methods. Iterated collocation and Galerkin methods. Error analysis.	<ul style="list-style-type: none"> <li>• Interactive exposure</li> <li>• Explanation</li> <li>• Conversation</li> <li>• Description</li> </ul>	
10. Nyström methods. Product integration methods. Error analysis. Discrete collocation and discrete Galerkin methods.	<ul style="list-style-type: none"> <li>• Interactive exposure</li> <li>• Explanation</li> <li>• Conversation</li> <li>• Description</li> </ul>	
11. Applications. Volterra's population model. Diffraction problems, Fresnel integrals.	<ul style="list-style-type: none"> <li>• Interactive exposure</li> <li>• Explanation</li> <li>• Conversation</li> <li>• Description</li> </ul>	
12. Applications to potential theory. The Thomas-Fermi equation.	<ul style="list-style-type: none"> <li>• Interactive exposure</li> <li>• Explanation</li> <li>• Conversation</li> <li>• Description</li> </ul>	
13. Applications to ocean waves. Green's function method for waves. Seismic response of dams.	<ul style="list-style-type: none"> <li>• Interactive exposure</li> <li>• Explanation</li> <li>• Conversation</li> <li>• Description</li> </ul>	
14. Heat transfer and heat radiation.	<ul style="list-style-type: none"> <li>• Interactive exposure</li> <li>• Explanation</li> <li>• Conversation</li> <li>• Description</li> </ul>	

### Bibliography

1. M. Rahman, Integral Equations and their Applications, WIT Press, Ashurst, Southampton, 2007.
2. A. M. Wazwaz, Linear and Nonlinear Integral Equations, Methods and Applications. Higher Education Press, Beijing. Springer, New York, 2011.
3. K. E. Atkinson, The Numerical Solution of Integral Equations of the Second Kind, Cambridge University Press, Cambridge, 1997.
4. S. Micula, G. V. Milovanović, Chapter 16: Iterative Processes and Integral Equations of the Second Kind, Book: Matrix and Operator Equations and Applications, Birkhäuser, Springer Nature, Heidelberg, 2023.
5. A. D. Polyanin, A. V. Manzhirov, Handbook of Integral Equations, 2nd ed., CRC Press, Boca Raton, 2008.

6. S. Prössdorf, B. Silbermann, Numerical Analysis for Integral and Related Operator Equations, Wiley, Oxford, 1991.		
8.2 Seminar / laboratory	Teaching methods	Remarks
1. Relationship between initial value/boundary value problems and integral equations.	<ul style="list-style-type: none"> <li>• Interactive exposure</li> <li>• Explanation</li> <li>• Conversation</li> <li>• Individual and group work</li> </ul>	The seminar is structured as 2 hours per week, every other week
2. Solvable integral equations.	<ul style="list-style-type: none"> <li>• Interactive exposure</li> <li>• Explanation</li> <li>• Conversation</li> <li>• Individual and group work</li> </ul>	
3. Volterra integral equations. Abel's integral equation.	<ul style="list-style-type: none"> <li>• Interactive exposure</li> <li>• Explanation</li> <li>• Conversation</li> <li>• Individual and group work</li> </ul>	
4. Fredholm integral equations. Mixed integral equations.	<ul style="list-style-type: none"> <li>• Interactive exposure</li> <li>• Explanation</li> <li>• Conversation</li> <li>• Individual and group work</li> </ul>	
5. Interpolation-based collocation and Galerkin methods. Iterated solutions.	<ul style="list-style-type: none"> <li>• Interactive exposure</li> <li>• Explanation</li> <li>• Conversation</li> <li>• Individual and group work</li> </ul>	
6. Nyström methods. Product integration. Discrete projection methods.	<ul style="list-style-type: none"> <li>• Interactive exposure</li> <li>• Explanation</li> <li>• Conversation</li> <li>• Individual and group work</li> </ul>	
7. Various applications.	<ul style="list-style-type: none"> <li>• Interactive exposure</li> <li>• Explanation</li> <li>• Conversation</li> <li>• Individual and group work</li> </ul>	
<b>Bibliography</b>		
1. M. Rahman, Integral Equations and their Applications, WIT Press, Ashurst, Southampton, 2007.		
2. A. M. Wazwaz, Linear and Nonlinear Integral Equations, Methods and Applications. Higher Education Press, Beijing. Springer, New York, 2011.		
3. K. E. Atkinson, The Numerical Solution of Integral Equations of the Second Kind, Cambridge University Press, Cambridge, 1997.		
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5. A. D. Polyanin, A. V. Manzhirov, Handbook of Integral Equations, 2nd ed., CRC Press, Boca Raton, 2008.		
6. S. Prössdorf, B. Silbermann, Numerical Analysis for Integral and Related Operator Equations, Wiley, Oxford, 1991.		

### 9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

<ul style="list-style-type: none"> <li>• Courses with similar content exist in the studying program of major universities in Romania and abroad, for Mathematics and Applied Mathematics students at the Master's level;</li> <li>• The knowledge and skills acquired in this course give students a foundation for launching a career in scientific research;</li> <li>• The analysis and modeling abilities acquired in this course are useful in any career path students may</li> </ul>
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choose.

## 10. Evaluation

Type of activity	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Share in the grade (%)
10.4 Course	- acquire the basic principles and notions in Integral Equations theory; - apply correctly various course concepts and methods	<b>Written exam</b>	70%
10.5 Seminar/lab activities	- understand and be able to use theory and results in problems and applications; - apply numerical procedures and algorithms to solve practical and real-life problems	- active participation in discussing and solving problems throughout the semester - individual presentation of solutions	30%
10.6 Minimum performance standards			
A grade of 5 or above (on a scale from 1 to 10) on <b>each</b> of the activities mentioned above (written exam, seminar evaluation)			

Date

23.04.2024

Signature of course coordinator

Prof. Sanda Micula, PhD. Habil.

Signature of seminar coordinator

Prof. Sanda Micula, PhD. Habil.

Date of approval

Signature of the head of department

Prof. dr. Mărcuș Andrei