#### **SYLLABUS**

1.1 Higher education		
institution	Babeş-Bolyai University Cluj-Napoca	
1.2 Faculty	Faculty of Mathematics and Computer Science	
1.3 Department	Department of Mathematics	
1.4 Field of study	Mathematics	
1.5 Study cycle	Master	
1.6 Study programme /	Advanced Methometics	
Qualification	Advanced Mathematics	

## 1. Information regarding the programme

## 2. Information regarding the discipline

2.1 Name of the discipline (en)		Integral Equations with Applications					
(ro)		Ecuații integrale cu aplicații					
2.2 Course coordinator		Prof. Sanda Micula, PhD. Habil.					
2.3 Seminar coordinator		Prof. Sanda Micula, PhD. Habil.					
2.4. Year of study	2	2.5 Semester	3	2.6. Type of evaluation	Ε	2.7 Type of discipline	DS Optional
2.8 Code of the discipline		MME3160					

## 3. Total estimated time (hours/semester of didactic activities)

3.1 Hours per week	3	Of which:	3.2 course	2	3.3 seminar/laboratory	1 sem
3.4 Total hours in the curriculum	42	Of which:	3.5 course	28	3.6 seminar/laboratory	14
Time allotment:					hours	
Learning using manual, course support, bibliography, course notes					40	
Additional documentation (in libraries, on electronic platforms, field documentation)					30	
Preparation for seminars/labs, homework, papers, portfolios and essays					40	
Tutorship					14	
Evaluations					9	
Other activities:						
3.7 Total individual study hours 133					<u> </u>	
3.8 Total hours per semester 175						
3.9 Number of ECTS credits 7						

## 4. Prerequisites (if necessary)

4.1. curriculum	Mathematical Analysis, Numerical Analysis
4.2. competencies	Knowledge of basic notions of operator theory
	Average programming skills

## 5. Conditions (if necessary)

5.1. for the course	Classroom with large blackboard and video projector
5.2. for the seminar /lab	Classroom with large blackboard and video projector/computers with
activities	Matlab

## 6. Specific competencies acquired

<b>Professional</b> competencies	<ul> <li>Ability to understand and manipulate concepts, results and advanced mathematical theories.</li> <li>Ability to model and analyze from the mathematical point of view real processes from other sciences, economics, and engineering.</li> <li>Ability to use the scientific language and to write scientific reports and papers.</li> </ul>
<b>Transversal</b> competencies	<ul> <li>Ability to inform themselves, to work independently or in a team in order to realize studies and to solve complex problems.</li> <li>Ability for continuous self-perfecting and study.</li> <li>Ability to use advanced and complementary knowledge in order to obtain a PhD in Pure Mathematics and Applied Mathematics.</li> </ul>

# 7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	<ul> <li>Acquire knowledge of the general theory of integral equations, with focus on applications.</li> <li>Gain the ability to apply concepts and results from integral equations theory to specific problems.</li> </ul>
7.2 Specific objective of the discipline	<ul> <li>Understand and be able to use main concepts and results from general integral equations theory.</li> <li>Be able to analyze the solvability of specific integral equations arising in applications.</li> <li>Understand, use and be able to derive numerical methods for the approximate solution of integral equations arising in applications from various fields.</li> </ul>

# 8. Content

8.1 Course	Teaching methods	Remarks
<b>1. Introduction</b> . Basic concepts. History of integral equations. Classifications and examples.	<ul> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Description</li> </ul>	
<b>2.</b> Types of integral equations with exact solutions.	<ul> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Description</li> </ul>	
<b>3.</b> Relationship between initial value/boundary value problems and integral equations.	<ul> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Description</li> </ul>	
<b>4. Volterra integral equations</b> . The method of successive approximations. Laplace transforms. Adomian decomposition.	<ul> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Description</li> </ul>	

<b>5.</b> Series solution. Volterra integral equations of	Interactive exposure
the first kind. Integral equations of the convolution	Explanation
type. Abel integral equation.	Conversation
type. Moer integral equation.	Description
6. Fredholm integral equations. The method of	Interactive exposure
successive approximations, Neumann series.	• Explanation
Adomian decomposition. Compact integral	Conversation
operators. Properties. The Fredholm alternative	Description
theorem.	1
7. Homogeneous Fredholm equations. Fredholm	Interactive exposure
integral equations of the first kind.	Explanation
	Conversation
	Description
8. Numerical methods. Degenerate kernel	Interactive exposure
methods. Taylor series approximation.	Explanation
Interpolatory degenerate kernel approximation.	Conversation
1 7 5 11	Description
9. Projection methods, collocation and Galerkin	Interactive exposure
methods. Iterated collocation and Galerkin	Explanation
methods. Error analysis.	Conversation
-	Description
10. Nyström methods. Product integration	Interactive exposure
methods. Error analysis. Discrete collocation and	• Explanation
discrete Galerkin methods.	Conversation
	Description
<b>11. Applications</b> . Volterra's population model.	• Interactive exposure
Diffraction problems, Fresnel integrals.	Explanation
	Conversation
	Description
<b>12.</b> Applications to potential theory. The Thomas-	• Interactive exposure
Fermi equation.	Explanation
	Conversation
	Description
<b>13.</b> Applications to ocean waves. Green's function	Interactive exposure
method for waves. Seismic response of dams.	Explanation
	Conversation
	Description
<b>14.</b> Heat transfer and heat radiation.	Interactive exposure
	• Explanation
	Conversation
Bibliography	Description

## Bibliography

1. M. Rahman, Integral Equations and their Applications, WIT Press, Ashurst, Southampton, 2007.

2. A. M. Wazwaz, Linear and Nonlinear Integral Equations, Methods and Applications. Higher Education Press, Beijing. Springer, New York, 2011.

3. K. E. Atkinson, The Numerical Solution of Integral Equations of the Second Kind, Cambridge University Press, Cambridge,1997.

4. S. Micula, G. V. Milovanović, Chapter 16: Iterative Processes and Integral Equations of the Second Kind, Book: Matrix and Operator Equations and Applications, Birkhäuser, Springer Nature, Heidelberg, 2023.
5. A. D. Polyanin, A. V. Manzhirov, Handbook of Integral Equations, 2nd ed., CRC Press, Boca Raton, 2008.

I. Relationship between initial value/boundary value problems and integral equations.       Interactive exposure       The seminar is structured as 2 hours per week, every other week         2. Solvable integral equations.       Interactive exposure       Explanation         3. Volterra integral equations.       Interactive exposure       Explanation         4. Fredholm integral equations.       Interactive exposure       Explanation         4. Fredholm integral equations.       Mixed integral       Interactive exposure         equations.       Explanation       Conversation         5. Interpolation-based collocation and Galerkin methods. Iterated solutions.       Interactive exposure       Explanation         6. Nyström methods. Product integration.       Discrete rojection methods.       Interactive exposure       Explanation         7. Various applications.       Interactive exposure       Explanation       Conversation         Individual and group work       Interactive exposure       Explanation         6. Nyström methods. Product integration. Discrete rojection methods.       Interactive exposure       Explanation         7. Various applications.       Interactive exposure       Explanation         9. Conversation       Individual and group work       Interactive exposure         10. Rescue exposure       Explanation       Conversation	6. S. Prössdorf, B. Silbermann, Numerical Analysis Oxford, 1991.	for Integral and Related Operator Equ	uations, Wiley,	
value problems and integral equations.Explanationstructured as 2 hours per week, every other week2. Solvable integral equations.Interactive exposureExplanation2. Solvable integral equations.Interactive exposureExplanation3. Volterra integral equations. Abel's integral equation.Interactive exposureExplanation4. Fredholm integral equations. Mixed integral equations.Interactive exposureExplanation5. Interpolation-based collocation and Galerkin nethods. Iterated solutions.Interactive exposureExplanation6. Nyström methods. Product integration. Discrete projection methods.Interactive exposureExplanation7. Various applications.Interactive exposureExplanation7. Various applications.Interactive exposureExplanation6. Nyström methods.Integration. Discrete registrian function e Explanation e Conversation e Individual and group workInteractive exposure7. Various applications.Interactive exposure e Explanation e Conversation e Explanation e ConversationExplanation e Conversation e Interactive exposure e Explanation e Conversation e Explanation e Conversation7. Various applications.Interactive exposure e Explanation e Conversation9. Nyström methods.Explanation e Conversation e Explanation e Conversation9. Nyström methods.Interactive exposure e Explanation e Conversation9. Oversation e Explanation e ConversationExplanation e Conversation9. Oversation e Explanation e ConversationExpl	8.2 Seminar / laboratory	Teaching methods	Remarks	
value problems and integral equations.Explanationstructured as 2 hours per week, every other week2. Solvable integral equations.Interactive exposureExplanation2. Solvable integral equations.Interactive exposureExplanation3. Volterra integral equations. Abel's integral equation.Interactive exposureExplanation4. Fredholm integral equations. Mixed integral equations.Interactive exposureExplanation5. Interpolation-based collocation and Galerkin nethods. Iterated solutions.Interactive exposureExplanation6. Nyström methods. Product integration. Discrete rojection methods.Interactive exposureExplanation7. Various applications.Interactive exposureExplanation7. Various applications.Interactive exposureExplanation6. Replanation represented balanceInteractive exposureExplanation7. Various applications.Interactive exposureExplanation7. Various applications.Interactive exposureExplanation7. Various applications.ExplanationConversation8. Negret applications.Interactive exposureExplanation9. Conversation Individual and group workInteractive exposureExplanation9. Conversation Individual and group workExplanationExplanation9. Conversation Individual and group workExplanationExplanation9. Conversation Individual and group workExplanationExplanation9. Conversation Individual and group workExplanationExplanation <td>1. Relationship between initial value/boundary</td> <td>· · · · · · · · · · · · · · · · · · ·</td> <td>The seminar is</td>	1. Relationship between initial value/boundary	· · · · · · · · · · · · · · · · · · ·	The seminar is	
Conversation     Individual and group work     Individual and group work     Interactive exposure     Explanation     Conversation     Individual and group work     S. Volterra integral equations. Abel's integral     squation.     Volterra integral equations. Abel's integral     squation.     S. Volterra integral equations. Mixed integral     squations.     Individual and group work     S. Interpolation-based collocation and Galerkin     nethods. Iterated solutions.     S. Nyström methods. Product integration. Discrete     roojection methods.     You applications.     Individual and group work     S. Nyström methods.     Product integration. Discrete     roojection methods.     You applications.     Individual and group work     S. Various applications.	value problems and integral equations.	Explanation	structured as 2	
2. Solvable integral equations.       Interactive exposure         2. Solvable integral equations.       Interactive exposure         3. Volterra integral equations. Abel's integral       Interactive exposure         equation.       Interactive exposure         equation.       Explanation         6. Fredholm integral equations. Mixed integral       Interactive exposure         equations.       Explanation         equations.       Interactive exposure         explanation       Conversation         Individual and group work       Interactive exposure         explanation.       Explanation         conversation       Individual and group work         4. Fredholm integral equations. Mixed integral equations.       Interactive exposure         explanation       Conversation         explanation       Conversation         equations.       Interactive exposure         explanation       Explanation         conversation       Individual and group work         5. Interpolation-based collocation and Galerkin       Interactive exposure         explanation       Explanation         6. Nyström methods. Product integration. Discrete projection methods.       Interactive exposure         explanation       Explanation         7. Various applica		1	hours per week,	
2. Solvable integral equations. <ul> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Individual and group work</li> </ul> 3. Volterra integral equations. Abel's integral equation. <ul> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Individual and group work</li> </ul> 4. Fredholm integral equations. Mixed integral equations. <ul> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Individual and group work</li> </ul> 5. Interpolation-based collocation and Galerkin methods. Iterated solutions. <ul> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Individual and group work</li> </ul> 6. Nyström methods. Product integration. Discrete projection methods. <ul> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Individual and group work</li> </ul> 7. Various applications. <ul> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Individual and group work</li> </ul>		• Individual and group work	every other week	
<ul> <li>Conversation         <ul> <li>Individual and group work</li> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Individual and group work</li> </ul> </li> <li>Fredholm integral equations. Mixed integral equations. Mixed integral</li> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Individual and group work</li> </ul> <li>Interpolation-based collocation and Galerkin nethods. Iterated solutions.</li> <li>Interpolation-based collocation and Galerkin</li> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Individual and group work</li> <li>Nyström methods. Product integration. Discrete projection methods.</li> <li>Nyström methods.</li> <li>Various applications.</li> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Individual and group work</li> <li>Various applications.</li>	<b>2.</b> Solvable integral equations.			
Individual and group work3. Volterra integral equations. Abel's integral equation.Interactive exposure Explanation Conversation Individual and group work4. Fredholm integral equations. Mixed integral equations.Interactive exposure Explanation Conversation Individual and group work5. Interpolation-based collocation and Galerkin nethods. Iterated solutions.Interactive exposure Explanation Conversation Individual and group work6. Nyström methods. Product integration. Discrete projection methods.Interactive exposure Explanation Conversation Explanation Conversation Explanation Conversation Individual and group work7. Various applications.Interactive exposure Explanation Conversation Explanation Conversation Explanation Conversation Explanation Conversation Explanation Conversation Explanation Conversation Explanation Conversation Explanation Conversation Conversation Explanation Conversation Explanation Conversation Explanation Conversation		Explanation		
3. Volterra integral equations. Abel's integral equation.       Interactive exposure         Sequation.       Explanation         4. Fredholm integral equations. Mixed integral equations.       Interactive exposure         equations.       Interactive exposure         equations.       Explanation         6. Fredholm integral equations. Mixed integral equations.       Interactive exposure         equations.       Explanation         6. Interpolation-based collocation and Galerkin methods. Iterated solutions.       Interactive exposure         6. Nyström methods. Product integration. Discrete projection methods.       Interactive exposure         6. Nyström methods.       Product integration. Discrete projection methods.         7. Various applications.       Interactive exposure         6. Nyström methods.       Explanation         6. Nyström methods.       Discrete         6. Nyström methods.       Interactive exposure         6. Nyström methods.       Explanation         6. Nyström methods.       Interactive exposure         7. Various applications.       Interactive exposure         6. Explanation       Explanation         6. Conversation       Individual and group work		Conversation		
equation.Explanation Conversation Individual and group work4. Fredholm integral equations. Mixed integral equations.Interactive exposure Explanation Conversation Individual and group work5. Interpolation-based collocation and Galerkin methods. Iterated solutions.Interactive exposure Explanation Conversation Individual and group work6. Nyström methods. Product integration. Discrete projection methods.Interactive exposure Explanation Conversation Individual and group work7. Various applications.Interactive exposure Explanation Explanation Conversation Individual and group work		Individual and group work		
Conversation     Individual and group work     Interactive exposure     Explanation     Conversation     Individual and group work     Interactive exposure     Explanation     Individual and group work     Interactive exposure     Explanation     Conversation     Individual and group work     Interactive exposure     Explanation     Conversation     Individual and group work     Interactive exposure     Explanation     Conversation     Individual and group work     Interactive exposure     Explanation     Individual and group work     Interactive exposure     Explanation     Individual and group work     Interactive exposure     Explanation     Individual and group work	<b>3.</b> Volterra integral equations. Abel's integral	Interactive exposure		
Individual and group workInteractive exposureequations.Equations.Interactive exposureExplanationConversationIndividual and group workInterpolation-based collocation and GalerkinInteractive exposureInteractive exposureExplanationConversationIndividual and group workInteractive exposureIndividual and group workIndividual and group workIndividual and group workIndividual and group workIndividual and group workInteractive exposureExplanationIndividual and group workInteractive exposureExplanationIndividual and group workInteractive exposureIndividual and group workIndividual an	equation.	Explanation		
4. Fredholm integral equations. Mixed integral equations.• Interactive exposure • Explanation • Individual and group work6. Interpolation-based collocation and Galerkin nethods. Iterated solutions.• Interactive exposure • Interactive exposure • Explanation • Conversation • Individual and group work6. Nyström methods. Product integration. Discrete projection methods.• Interactive exposure • Explanation • Interactive exposure • Interactive exposure • Interactive exposure • Individual and group work7. Various applications.• Interactive exposure • Explanation • Individual and group work		Conversation		
equations.Explanation6. Interpolation-based collocation and Galerkin methods. Iterated solutions.Interactive exposure Explanation Conversation Individual and group work6. Nyström methods. Product integration. Discrete porojection methods.Interactive exposure Explanation Explanation Individual and group work7. Various applications.Interactive exposure Explanation Conversation Individual and group work		Individual and group work		
<ul> <li>Conversation</li> <li>Individual and group work</li> <li>Interpolation-based collocation and Galerkin methods. Iterated solutions.</li> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Individual and group work</li> <li>Nyström methods. Product integration. Discrete projection methods.</li> <li>Nyström methods.</li> <li>Individual and group work</li> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Individual and group work</li> <li>Individual and group work</li> <li>Substration</li> <li>Conversation</li> <li>Individual and group work</li> <li>Individual and group work</li> <li>Individual and group work</li> <li>Conversation</li> <li>Individual and group work</li> <li>Conversation</li> <li>Conversation</li> <li>Conversation</li> <li>Conversation</li> <li>Conversation</li> <li>Conversation</li> <li>Conversation</li> <li>Individual and group work</li> </ul>	4. Fredholm integral equations. Mixed integral	1		
Individual and group work5. Interpolation-based collocation and Galerkin methods. Iterated solutions.• Interactive exposure • Explanation • Conversation • Individual and group work6. Nyström methods. Product integration. Discrete brojection methods.• Interactive exposure • Explanation 	equations.	Explanation		
<ul> <li>5. Interpolation-based collocation and Galerkin nethods. Iterated solutions.</li> <li>6. Nyström methods. Product integration. Discrete projection methods.</li> <li>7. Various applications.</li> <li>9. Interactive exposure</li> <li>9. Interactive exposure</li> <li>9. Interactive exposure</li> <li>9. Interactive exposure</li> <li>9. Explanation</li> <li>9. Conversation</li> <li>9. Interactive exposure</li> <li>9. Explanation</li> <li>9. Conversation</li> <li>9. Interactive exposure</li> <li>9. Explanation</li> <li>9. Conversation</li> <li>9. Interactive exposure</li> <li>9. Explanation</li> <li>9. Interactive exposure</li> <li>9. Explanation</li> <li>9. Conversation</li> <li>9. Interactive exposure</li> <li>9. Explanation</li> <li>9. Conversation</li> <li>9. Explanation</li> <li>9. Conversation</li> <li>9. Explanation</li> <li>9. Conversation</li> <li< td=""><td></td><td>Conversation</td><td></td></li<></ul>		Conversation		
nethods. Iterated solutions.Explanation• Explanation• Conversation• Individual and group work• Individual and group work• Nyström methods. Product integration. Discrete projection methods.• Interactive exposure • Explanation • Conversation • Individual and group work• Various applications.• Interactive exposure • Explanation • Conversation • Explanation • Conversation • Explanation • Conversation • Explanation		Individual and group work		
<ul> <li>Conversation         <ul> <li>Conversation</li> <li>Individual and group work</li> </ul> </li> <li>Nyström methods. Product integration. Discrete         <ul> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Conversation</li> <li>Individual and group work</li> </ul> </li> <li>Various applications.</li> <li>Interactive exposure</li> <li>Explanation</li> <li>Explanation</li> <li>Conversation</li> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> </ul>	5. Interpolation-based collocation and Galerkin	Interactive exposure		
<ul> <li>Individual and group work</li> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Individual and group work</li> </ul> 7. Various applications. <ul> <li>Interactive exposure</li> <li>Explanation</li> <li>Interactive exposure</li> <li>Explanation</li> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> </ul>	methods. Iterated solutions.	Explanation		
6. Nyström methods. Product integration. Discrete projection methods.       • Interactive exposure         • Explanation       • Conversation         • Individual and group work       • Interactive exposure         7. Various applications.       • Interactive exposure         • Explanation       • Explanation         • Onversation       • Interactive exposure         • Explanation       • Onversation		Conversation		
brojection methods.       • Explanation         • Conversation       • Individual and group work         7. Various applications.       • Interactive exposure         • Explanation       • Explanation         • Conversation       • Conversation		Individual and group work		
Conversation     Individual and group work      Interactive exposure     Explanation     Conversation	6. Nyström methods. Product integration. Discrete	1		
<ul> <li>Individual and group work</li> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> </ul>	projection methods.	1		
<ul> <li>7. Various applications.</li> <li>Interactive exposure</li> <li>Explanation</li> <li>Conversation</li> </ul>		Conversation		
Explanation     Conversation		Individual and group work		
Conversation	7. Various applications.	1		
Individual and group work				
		Individual and group work		

### Bibliography

1. M. Rahman, Integral Equations and their Applications, WIT Press, Ashurst, Southampton, 2007.

2. A. M. Wazwaz, Linear and Nonlinear Integral Equations, Methods and Applications. Higher Education Press, Beijing. Springer, New York, 2011.

3. K. E. Atkinson, The Numerical Solution of Integral Equations of the Second Kind, Cambridge University Press, Cambridge,1997.

4. S. Micula, G. V. Milovanović, Chapter 16: Iterative Processes and Integral Equations of the Second Kind, Book: Matrix and Operator Equations and Applications, Birkhäuser, Springer Nature, Heidelberg, 2023.
5. A. D. Polyanin, A. V. Manzhirov, Handbook of Integral Equations, 2nd ed., CRC Press, Boca Raton, 2008.

6. S. Prössdorf, B. Silbermann, Numerical Analysis for Integral and Related Operator Equations, Wiley, Oxford, 1991.

# 9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program

- Courses with similar content exist in the studying program of major universities in Romania and abroad, for Mathematics and Applied Mathematics students at the Master's level;
- The knowledge and skills acquired in this course give students a foundation for launching a career in scientific research;
- The analysis and modeling abilities acquired in this course are useful in any career path students may

## 10. Evaluation

Type of activity	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Share in the grade (%)		
10.4 Course	<ul> <li>acquire the basic</li> <li>principles and notions in</li> <li>Integral Equations theory;</li> <li>apply correctly various</li> <li>course concepts and</li> <li>methods</li> </ul>	Written exam	70%		
10.5 Seminar/lab activities		<ul> <li>active participation in discussing and solving problems throughout the semester</li> <li>individual presentation of solutions</li> </ul>	30%		
10.6 Minimum performance standards					
1		of the activities mentioned abo	ove (written exam,		

Date	Signature of course coordinator	Signature of seminar coordinator
23.04.2024	Prof. Sanda Micula, PhD. Habil.	Prof. Sanda Micula, PhD. Habil.

Date of approval

Signature of the head of department

Prof. dr. Mărcuş Andrei