

## SYLLABUS

### 1. Information regarding the programme

1.1 Higher education institution	<b>Babeş Bolyai University</b>
1.2 Faculty	<b>Faculty of Mathematics and Computer Science</b>
1.3 Department	<b>Doctoral School of Mathematics and Computer Science</b>
1.4 Field of study	<b>Mathematics</b>
1.5 Study cycle	<b>Doctoral studies</b>
1.6 Study programme / Qualification	<b>TRAINING PROGRAMME BASED ON ADVANCED UNIVERSITY STUDIES</b>

### 2. Information regarding the discipline

2.1 Name of the discipline	<b>MDE3145 Representation theory of finite groups</b>						
2.2 Course coordinator	prof. dr. Andrei Marcus						
2.3 Seminar coordinator	prof. dr. Andrei Marcus						
2.4. Year of study	<b>1</b>	2.5 Semester	<b>1</b>	2.6. Type of evaluation	<b>E</b>	2.7 Type of discipline	<b>Optional</b>

### 3. Total estimated time (hours/semester of didactic activities)

3.1 Hours per week	3	Of which: 3.2 course	2	3.3 seminar/laboratory	1
3.4 Total hours in the curriculum	36	Of which: 3.5 course	24	3.6 seminar/laboratory	12
Time allotment:	hours				
Learning using manual, course support, bibliography, course notes	54				
Additional documentation (in libraries, on electronic platforms, field documentation)	50				
Preparation for seminars/labs, homework, papers, portfolios and essays	60				
Tutorship	10				
Evaluations	10				
Other activities: project	30				
3.7 Total individual study hours	214				
3.8 Total hours per semester	250				
3.9 Number of ECTS credits	10				

### 4. Prerequisites (if necessary)

4.1. curriculum	<ul style="list-style-type: none"> <li>- deep knowledge of bachelor level algebra, especially of the following subjects:</li> <li>- algebraic structures</li> <li>- linear algebra</li> </ul>
4.2. competencies	<ul style="list-style-type: none"> <li>- ability to perform symbolic calculations ability to operate with abstract concepts</li> <li>- ability to do logical deductions</li> <li>- ability to solve mathematics problems bases on aquired notions</li> </ul>

## 5. Conditions (if necessary)

5.1. for the course	<ul style="list-style-type: none"> <li>• blackboard, projector</li> </ul>
5.2. for the seminar /lab activities	<ul style="list-style-type: none"> <li>• blackboard</li> </ul>

## 6. Specific competencies acquired

<b>Professional competencies</b>	<ul style="list-style-type: none"> <li>• ability to perform symbolic calculations in various structures (groups, rings and fields, vector spaces, algebras, matrix algebras etc)</li> <li>• ability to operate with abstract concepts</li> <li>• ability to complex logical deductions</li> <li>• ability to solve mathematics problems bases on aquired notions</li> </ul>
<b>Transversal competencies</b>	<ul style="list-style-type: none"> <li>- abstract reasoning</li> <li>- applying mathematics in real life</li> <li>- ability to solve problems</li> </ul>

## 7. Objectives of the discipline (outcome of the acquired competencies)

7.1 General objective of the discipline	<ul style="list-style-type: none"> <li>• Advanced knowledge on group theory. Ability to solve more difficult problems</li> </ul>
7.2 Specific objective of the discipline	<ul style="list-style-type: none"> <li>• students will operate with fundamental concepts of group theory</li> <li>• students will aquire knowlegde regarding the structure of groups from various important classes.</li> <li>• students solve problems, theoretical and practical, using instruments of modern algebra, regarding matrix representations and characters.</li> </ul>

## 8. Content

8.1 Course	Teaching methods	Remarks
Week 1. Algebras, subalgebras, homomorphisms, ideals, factor algebras. Examples. Group algebra.	Explanation, dialogue, examples, proofs	
Week 2. Representations and modules. Simple modules (irreducible representations) and indecomposable modules.	Explanation, dialogue, examples, proofs	
Week 3. Tensor products. Hopf algebras.	Explanation, dialogue, examples, proofs	
Week 4. Semisimple algebras and modules. The Jordan-Holder and Krull-Schmidt Theorems.	Explanation, dialogue, examples, proofs	
Week 5. Representations of finite groups. Characters. Orthogonality. Character table computations.	Explanation, dialogue, examples, proofs	
Week 6. Products of characters. Induced characters. Frobenius reciprocity.	Explanation, dialogue, examples, proofs	
Week 7. Burnside's Theorem.	Explanation, dialogue, examples, proofs	
Week 8. Group algebras over fields of characteristic $p > 0$ and over discrete valuation rings	Explanation, dialogue, examples, proofs	
Week 9. Modular characters.	Explanation, dialogue, examples, proofs	

Week 10. Representations of the symmetric group.	Explanation, dialogue, examples, proofs	
Week 11. Clifford's Theorems. Projective representations	Explanation, dialogue, examples, proofs	
Week 12. G-algebras and crossed products	Explanation, dialogue, examples, proofs	
Bibliography		
[1] J.L. Alperin and R.B. Bell. <i>Groups and representations</i> . Springer-Verlag. 1995.		
[2] P. Etingof et al. <i>Introduction to representation theory</i> . American Mathematical Society 2011.		
8.2 Seminar / laboratory	Teaching methods	Remarks
Week 1. Algebras, subalgebras, homomorphisms, ideals, factor algebras. Examples. Group algebra.	dialogue, examples, proofs	
Week 2. Representations and modules. Simple modules (irreducible representations) and indecomposable modules.	dialogue, examples, proofs	
Week 3. Tensor products. Hopf algebras.	dialogue, examples, proofs	
Week 4. Semisimple algebras and modules. The Jordan-Holder and Krull-Schmidt Theorems.	dialogue, examples, proofs	
Week 5. Representations of finite groups. Characters. Orthogonality. Character table computations.	dialogue, examples, proofs	
Week 6. Products of characters. Induced characters. Frobenius reciprocity.	dialogue, examples, proofs	
Week 7. Burnside's Theorem.	dialogue, examples, proofs	
Week 8. Group algebras over fields of characteristic $p > 0$ and over discrete valuation rings	dialogue, examples, proofs	
Week 9. Burnside's Theorem.	dialogue, examples, proofs	
Week 10. Representations of the symmetric group.	dialogue, examples, proofs	
Week 11. Clifford's Theorems. Projective representations	dialogue, examples, proofs	
Week 12. G-algebras and crossed products	dialogue, examples, proofs	
Bibliography		
3. B.E. Sagan. <i>The symmetric group</i> . Springer-Verlag. 2001.		
4. I. Assem. <i>Algebras et modules</i> . Univ. Ottawa. 1997.		
5. T.Y. Lam. <i>A first course in noncommutative rings</i> . 2nd ed. Springer Verlag 2001.		
6. M. Auslander, I. Reiten, S.O. Smalø. <i>Representation Theory of Artin Algebras</i> , Cambridge Univ. Press, 1995.		
7. D.J. Benson, <i>Representations and Cohomology, vol. I, II</i> . Cambridge Univ. Press, 1998.		

**9. Corroborating the content of the discipline with the expectations of the epistemic community, professional associations and representative employers within the field of the program**

- Such a course exists in the curricula of many major universities;
- Groups are fundamental mathematical structures and have multiple applications in geometry, number theory, cryptography, chemistry and physics, as they measure symmetry.

## 10. Evaluation

Type of activity	10.1 Evaluation criteria	10.2 Evaluation methods	10.3 Share in the grade (%)
10.4 Course	- know the basic principles of the field; - apply the new concepts	- written exam	75%
10.5 Seminar/lab activities	- problem solving	- homeworks	25%
10.6 Minimum performance standards			
➤ to aquire 5 points to pass the exam			

Date

30.06.2021

Signature of course coordinator

Prof.dr. Andrei Mărcuș

Signature of seminar coordinator

Prof.dr. Andrei Mărcuș

Date of approval

07.07.2021

Signature of the head of doctoral school

Prof. dr. Gabriela Czibula