BABEŞ-BOLYAI UNIVERSITY CLUJ-NAPOCA
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

MATE-INFO UBB 2025 Contest
Written exam in MATHEMATICS

1. If the function $f\colon (1,+\infty)\to\mathbb{R}$ is defined by the expression

$$f(x)=\log_x\frac{1}{2}+\log_2\frac{1}{x},\forall x>1,$$

then the value of $f(8)$ is

A -6; B $-\frac{8}{3}$; C $-\frac{10}{3}$; D $\frac{8}{3}$.

2. If S is the set of real solutions of the equation $|x+1|+|x-1|=4$, then the sum of all the elements in S is

A 2; B 4; C -4; D 0.

3. The point $A(2,3)$ is the vertex of a triangle ABC whose orthocenter is the point $H(1,-1)$. Which of the following statements are true?

A The slope of the line AH is $\frac{1}{4}$. B The slope of the line AH is 4.

C The slope of the line BC is -4. D The slope of the line BC is $-\frac{1}{4}$.

4. The value of $\cos\frac{2025\pi}{6}$ is

A -1; B $-\frac{1}{2}$; C 0; D $\frac{1}{2}$.

5. Let $\vec{i}$ and $\vec{j}$ be the versors of the axes in a cartesian system. If the vectors $\vec{a}=2\vec{i}-x\vec{j}$ and $\vec{b}=x\vec{i}-8\vec{j}$ are collinear, then the value of the parameter $x\in\mathbb{R}$ can be:

A -4; B 0; C 4; D 8.

6. The terms of a geometric progression $a<b<c$ are natural numbers satisfying $a+b+c=21$. Which of the following statements are true, knowing that also the ratio of the progression is a natural number?

A There exists at least one geometric progression with this property.
B There exist exactly two geometric progressions with this property.
C The ratio of the progression is an even number.
D The ratio of the progression is an odd number.

7. For every $n\in\mathbb{N}^*$ we denote by $a_n=\lim\limits_{x\to\frac{\pi}{2}}\dfrac{1-\sin^n x}{\cos^2 x}$. The value of the limit $\lim\limits_{n\to\infty}\sqrt{n}\,(\sqrt{a_{n+1}}-\sqrt{a_n})$ is:

A $\frac{1}{2}$; B $\frac{1}{\sqrt{2}}$; C $\frac{1}{2\sqrt{2}}$; D $2\sqrt{2}$.

8. The value of the limit $\lim\limits_{x\to 0}\left(\dfrac{\operatorname{tg} x}{x}\right)^{1/x^2}$ is:

A 1; B $\mathrm{e}^{1/3}$; C $\mathrm{e}^{2/3}$; D $\mathrm{e}^{1/6}$.

9. Let $a, b \in \mathbb{R}$ and the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} -1 + \operatorname{arctg} x, & \text{if } x < 0 \\ ax + b, & \text{if } x \geq 0. \end{cases}$$

If f is differentiable on $\mathbb{R}$, then the value of the sum $a + b$ is:

A 0; B $\dfrac{\pi}{4}$; C $\dfrac{\pi}{2}$; D -2.

10. Consider the square $MNPQ$ with side length $MN = 2$. Which of the following statements are true?

A $\overrightarrow{MN} \cdot \overrightarrow{MQ} = 0$. B $\overrightarrow{MP} \cdot \overrightarrow{NQ} = 0$. C $\overrightarrow{MN} \cdot \overrightarrow{MP} = 4$. D $\overrightarrow{MN} \cdot \overrightarrow{MP} = 4\sqrt{2}$.

11. Let $ABCD$ be a quadrilateral and denote by G_1 and G_2 the centroids (centers of mass) of the triangles ABD and BCD, respectively. Denote by M the midpoint of the diagonal BD. Which of the following statements are true?

A $\overrightarrow{AG_1} = \dfrac{1}{2}\overrightarrow{AM}$. B $\overrightarrow{AG_1} = \dfrac{2}{3}\overrightarrow{AM}$. C $\overrightarrow{G_1G_2} = \dfrac{1}{3}\overrightarrow{AC}$. D $\overrightarrow{G_1G_2} = \dfrac{1}{2}\overrightarrow{AC}$.

12. Let $M = \left\{ z \in \mathbb{C}^* \mid z + \dfrac{2025}{z} \in \mathbb{R} \right\}$. Which of the following statements are true?

A $\mathbb{R}^* \subseteq M$.

B If $z \in M$, then $\dfrac{2025}{z} \in M$.

C If $z \in M \setminus \mathbb{R}^*$, then $|z|$ is uniquely determined.

D The set $M \setminus \mathbb{R}^*$ is empty.

13. The number of strictly positive integers m for which the second degree equation

$$mx^2 + 20x + m = 0$$

admits two distinct real solutions is

A 0; B 9; C 10; D 11.

14. Let S be the set of real solutions of the equation

$$3^{x-2} + 3^{x-1} + 3^x + 3^{x+1} = 2^{x+1} + 2^{x+3}.$$

Which of the following statements are true?

A S has exactly one element.

B There is at least one irrational number in S.

C S has exactly two elements.

D There is at least one rational number in S.

15. The value of the integral $\displaystyle\int_0^{\pi/3} \frac{\mathrm{d}x}{\cos x}$ is:

A $\ln(2 - \sqrt{3})$; B $\ln(2 + \sqrt{3})$; C $\ln(3 - \sqrt{3})$; D $\ln(4 + \sqrt{3})$.

16. The number of solutions of the equation $\sin^2(2x) + \cos^2(x) = 1$ in the interval $[0, \pi]$ is:

A 4; B 3; C 2; D 1.

17. Let $f : [0, \infty) \to \mathbb{R}$ be the function defined by

$$f(x) = \begin{cases} x, & \text{if } x \in [0, 1] \\ \dfrac{1}{x^2}, & \text{if } x > 1. \end{cases}$$

Given a real number $t \in (0, 1)$, the line with equation $y = t$ intersects the graph of f in the points A and B. Let A' and B' be the projections on the axis Ox of the points A and B, respectively. When t varies in $(0, 1)$, the maximal value of the area of the rectangle $AA'B'B$ is:

A $\dfrac{\sqrt[3]{2}}{2}$; B $\dfrac{3\sqrt[3]{2}}{8}$; C $\dfrac{\sqrt[3]{2}}{4}$; D $\dfrac{\sqrt[3]{2}}{8}$.

18. The lines with equations $11x + 3y - 7 = 0$ and $12x + y - 19 = 0$ intersect in the point M. The lines d_1 and d_2 pass through M and are situated at distance $\sqrt{2}$ from the point $A(3, -2)$. Which of the following statements are true?

A The sum of the coordinates of the point M is -7.
B The sum of the coordinates of the point M is -3.
C The lines d_1 and d_2 have equations: $7x + y - 9 = 0$ and $x - y - 7 = 0$.
D The lines d_1 and d_2 have equations: $x + 7y - 9 = 0$ and $x + y + 7 = 0$.

19. Consider the rhombus $ABCD$ with vertex $A(1, 0)$ and whose side AB is situated on the Ox axis. If the diagonals intersect in the point $M(3, 4)$, then the area of the rhombus is

A 20; B 40; C 60; D 80.

The problems **20**, **21** and **22** refer to the function $f : [0, 4] \to \mathbb{R}$, defined by $f(x) = (2 - x)\sqrt{4x - x^2}$.

20. Which of the following statements are true?

A The graph of f has an axis of symmetry parallel to the Oy axis.
B The graph of f has a center of symmetry.
C The graph of f has neither a center of symmetry nor an axis of symmetry parallel to the Oy axis.
D The graph of f has a center of symmetry but does not have an axis of symmetry parallel to the Oy axis.

21. The number of points of local extrema of f is:

A 0; B 2; C 3; D 4.

22. The value of the integral $\displaystyle\int_0^4 |f(x)|\,\mathrm{d}x$ is:

A $\dfrac{8}{3}$; B $\dfrac{4}{3}$; C $\dfrac{16}{3}$; D 4.

23. Let $a \in \mathbb{C}$ and consider the system of equations

$$\begin{cases} x + y - z = 1 \\ 2x + ay + z = 1 \\ 2x - y + az = 1. \end{cases}$$

Which of the following statements are true?

A For every $a \in \mathbb{R}$ the system admits a unique solution.
B If the system is incompatible, then $|a| = \sqrt{5}$.
C There exists $a \in \mathbb{C}$ for which the system admits infinitely many solutions.
D If for a solution (x, y, z) we have $x + y + z = 0$, then $a \notin \mathbb{R}$.

24. Let $f\colon (\mathbb{Z}_{10}, +) \to (\mathbb{Z}_5, +)$ be a group morphism. Which of the following statements are true?

A If $\mathrm{Im}(f)$ has a unique element, then this element is $\widehat{1}$.

B $\widehat{0} \in \mathrm{Im}(f)$.

C If $\mathrm{Im}(f)$ has at least two elements, then f is surjective.

D There exists one unique morphism $f\colon (\mathbb{Z}_{10}, +) \to (\mathbb{Z}_5, +)$.

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Correct Answers

BBU Math-CS Contest 2025

Written test in MATHEMATICS

1. **C**
2. **D**
3. **B**, **D**
4. **C**
5. **A**, **C**
6. **A**, **B**, **C**
7. **C**
8. **B**
9. **A**
10. **A**, **B**, **C**
11. **B**, **C**
12. **A**, **B**, **C**
13. **B**
14. **A**, **D**
15. **B**
16. **A**
17. **B**
18. **B**, **C**
19. **D**
20. **B**, **D**
21. **D**
22. **C**
23. **A**, **B**, **D**
24. **B**, **C**