BABEŞ-BOLYAI UNIVERSITY CLUJ-NAPOCA
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

ADMISSION 2024

Written exam in MATHEMATICS

IMPORTANT NOTE: Problems can have one or more correct answers, which the candidate should indicate on the test form. The grading system of the multiple choice exam can be found in the set of rules of the competition.

1. If $f : \mathbb{R} \to \mathbb{R}$,

$$f(x) = 3^{x+1} - 9^x,$$

then the value of the expression $f(\log_3 4)$ is

A 4; B -4; C -11; D -3.

2. In the parallelogram $ABCD$ we have $AB = 2$, $AD = 1$ and $\widehat{C} = 60°$. Which of the following statements are true?

A $BC = 1$; B $CD = 1$; C $AC = \sqrt{3}$; D $BD = \sqrt{3}$.

3. The value of the limit $\lim\limits_{n\to\infty} \sqrt{n}\left(\sqrt{n+2^n} - \sqrt{n}\right)$ is:

A 0; B 2; C 1; D $+\infty$.

4. Let A be a set with n elements. If the number of subsets of A with 2 elements is 21, then

A $n \in (2, 6]$;
B $n \in (6, 10]$;
C $n \in (10, 14]$;
D there are no such values of n.

5. Let S be the set of real solutions of the equation

$$4^x - 2^x \cdot 3^{x+1} = 4 \cdot 9^x.$$

Which of the following statements are true?

A S has exactly two elements;
B S has exactly one element;
C $\dfrac{2}{1 - \log_2 3} \in S$;
D $\dfrac{2}{1 + \log_2 3} \in S$.

6. In the triangle ABC we have $E \in (AB)$, $EB = 3 \cdot EA$, $F \in (AC)$ and $FC = 2 \cdot FA$. Knowing that the points A, E and F have coordinates $A(1,3)$, $E(3,6)$ and $F(4,18)$, the coordinates of the centroid (center of mass) G of the triangle ABC are:

[A] $G\left(\frac{31}{6}, \frac{89}{6}\right)$; [B] $G\left(\frac{20}{3}, 22\right)$; [C] $G\left(\frac{61}{18}, \frac{73}{6}\right)$; [D] $G\left(\frac{50}{9}, 22\right)$.

7. The points $D(2,1)$, $E(-2,5)$ and $F(1,4)$ are the midpoints of the sides AB, BC and AC of the triangle ABC. The area of ABC is:

[A] 2; [B] 4; [C] 8; [D] 16.

8. The value of the integral $\displaystyle\int_0^{\pi/3} \frac{\cos x}{1 + 4\sin^2 x}\, dx$ is:

[A] $\frac{\pi}{6}$; [B] $\frac{\pi}{3}$; [C] $\frac{\pi}{12}$; [D] $\frac{\pi}{2}$.

9. Given the real number a, the function $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = \dfrac{x^2 + ax + 1}{\sqrt{x^2+1}}$. Consider the points $A(0,1)$ and $B(2,7)$. The value of a for which the line AB is tangent to the graph of f in the point A is:

[A] 5; [B] 3; [C] 0; [D] -3.

10. In the parallelogram $ABCD$ we have $A(2,1)$, $B(4,3)$ and $C(7,2)$. The equation of the line BD is:

[A] $x - y - 1 = 0$; [B] $x + 3y - 13 = 0$; [C] $x - 5y + 3 = 0$; [D] $3x + y = 15$.

11. Let $\alpha \in (\pi, 2\pi)$ such that $\operatorname{tg}(\alpha) = \frac{1}{2}$. Which of the following statements are true?

[A] $\sin(\alpha) = -\frac{\sqrt{5}}{5}$; [B] $\cos(\alpha) = -\frac{2\sqrt{5}}{5}$; [C] $\sin(2\alpha) = -\frac{4}{5}$; [D] $\cos(2\alpha) = \frac{3}{5}$.

12. Let $ABCD$ be a square with side length 1. Which of the following statements are true?

[A] $\overrightarrow{AB} \cdot \overrightarrow{BC} = 0$; [B] $\overrightarrow{AB} \cdot \overrightarrow{CD} = 1$; [C] $\overrightarrow{AB} \cdot \overrightarrow{BD} = \frac{\sqrt{2}}{2}$; [D] $\overrightarrow{AB} \cdot \overrightarrow{DB} = \frac{\sqrt{2}}{2}$.

13. The integer numbers $b_1, b_2, b_3, \ldots, b_9, b_{10}$ are in a geometric progression with ratio $q = 2$ and

$$S = b_1 + b_2 + b_3 + \ldots + b_{10}.$$

Which of the following statements are correct?

[A] S is divisible by 11;
[B] if S is a perfect square, then b_1 is divisible by 31;
[C] if b_1 is odd, then S is even;
[D] if b_1 is odd, then S is odd.

14. Let a be a real parameter and consider the system of equations:

$$\begin{cases} x + 3y - z = 1 \\ -x - 2y + z = a \\ x + ay + 2z = -2. \end{cases}$$

Which of the following statements are correct?

- A There exists $a \in \mathbb{R}$ for which the determinant of the matrix of the system is 0.
- B For every $a \in \mathbb{R}$ the system has a unique solution.
- C If $a = 1$, then $x + y + 2z = 1$.
- D For every $a \in \mathbb{R}$ we have $x + y + 2z < 0$.

15. In the permutation group S_4 consider the elements

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 3 & 1 & 2 \end{pmatrix} \quad \text{and} \quad \tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}.$$

If $x \in S_4$ is a permutation such that $x\sigma = \tau$, then

- A x is not uniquely determined;
- B x is uniquely determined;
- C $x^2 = \tau$;
- D $x^2 = \sigma$.

16. Consider the functions $f, g : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = |x|$ and $g(x) = x$ for every $x \in \mathbb{R}$. Which of the following statements are true?

- A The function $f + g$ is continuous on $\mathbb{R}$;
- B The function $f + g$ is strictly monotonic on $\mathbb{R}$;
- C The function f is differentiable on $\mathbb{R}$;
- D The function $f \cdot g$ is differentiable at 0.

17. Let $a, b \in \mathbb{R}$ and the function $f : D \to \mathbb{R}$ defined by $f(x) = \dfrac{x}{ax^2 + bx + 8}$, where $D \subseteq \mathbb{R}$ is the maximal possible domain of definition for f. If $x = -2$ is a point of local extremum of f and the the line of equation $x = 2$ is a vertical asymptote for the graph of f, then the value of the sum $a + b$ is:

- A -6;
- B -10;
- C 10;
- D -2.

18. Let $m \in \mathbb{R}$ be a parameter, and $f \colon \mathbb{R} \to \mathbb{R}$ the function defined by $f(x) = (x^2 + mx)\mathrm{e}^{-x}$. Which of the following statements are true?

- A The graph of f has an asymptote towards $+\infty$;
- B If $m = 2024$, then the function f has a point of global maximum;
- C For all $m \in \mathbb{R}$, the function f has exactly two points of local extremum;
- D There exists $m \in \mathbb{R}$ for which the function f has exactly one point of local extremum.

19. Let $\overrightarrow{i}$ and $\overrightarrow{j}$ be the versors of a cartesian system. If the vectors $\overrightarrow{u} = (p+5)\overrightarrow{i} + 2\overrightarrow{j}$ and $\overrightarrow{v} = 4\overrightarrow{i} + 10\overrightarrow{j}$ are perpendicular, then the parameter $p \in \mathbb{R}$ can be:

A -10; B $-\frac{21}{5}$; C 0; D $\frac{29}{5}$.

20. In the triangle ABC we have $A(1,0)$, $B\left(5, \frac{4\sqrt{3}}{3}\right)$ and $\widehat{A} = 60°$. The equation of the line AC can be:

A $3y + \sqrt{3}x = \sqrt{3}$; B $3y - \sqrt{3}x = -\sqrt{3}$; C $y - \sqrt{3}x = -\sqrt{3}$; D $x = 1$.

21. For every matrix $X \in \mathcal{M}_3(\mathbb{R})$ we write $\mathrm{Tr}(X)$ for the sum of the elements on the main diagonal of X. If

$$A = \begin{pmatrix} 1 & 0 & 0 \\ -2 & -1 & 0 \\ -4 & -3 & -2 \end{pmatrix},$$

then the value of the expression $\mathrm{Tr}(A^3) + \det(A^3)$ is

A -1; B 1; C 0; D 2.

22. If x_1, x_2 and x_3 are the roots of the polynomial

$$f = X^3 + X^2 + 10X + 2$$

then the value of the expression $\frac{x_1}{x_2+x_3} + \frac{x_2}{x_1+x_3} + \frac{x_3}{x_1+x_2}$ is equal to

A 0; B $\frac{13}{8}$; C $\frac{13}{7}$; D $-\frac{13}{8}$.

23. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = \frac{x^2}{1+\mathrm{e}^x}$. The area of the set in the plane enclosed by the graph of f, the Ox axis and the two lines of equations $x = -1$ and $x = 1$ is:

A 0; B $\frac{2}{3}$; C $\frac{1}{3}$; D $\frac{1}{6}$.

24. The value of the limit $\lim\limits_{x\to+\infty} \left(1 - \cos\frac{1}{x}\right)^{1/\ln x}$ is:

A e; B $\frac{1}{\mathrm{e}}$; C $\frac{1}{\mathrm{e}^2}$; D $\frac{1}{\sqrt{\mathrm{e}}}$.

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Correct Answers

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1. **B**
2. **A**, **D**
3. **D**
4. **B**
5. **B**, **C**
6. **B**
7. **D**
8. **A**
9. **B**
10. **D**
11. **A**, **B**, **D**
12. **A**
13. **A**, **B**, **D**
14. **B**, **D**
15. **B**
16. **A**, **D**
17. **A**
18. **A**, **C**
19. **A**
20. **A**, **D**
21. **C**
22. **D**
23. **C**
24. **C**