BABEŞ-BOLYAI UNIVERSITY CLUJ-NAPOCA
FACULTY OF MATHEMATICS AND COMPUTER SCIENCE

ADMISSION 2024

Written exam in MATHEMATICS

IMPORTANT NOTE: Problems can have one or more correct answers, which the candidate should indicate on the test form. The grading system of the multiple choice exam can be found in the set of rules of the competition.

1. If $f\colon \mathbb{R} \to \mathbb{R}$,

$$f(x) = 4^x - 2^{x+1},$$

then the value of the expression $f(\log_2 3)$ is

A -1; B 2; C 3; D 5.

2. The value of the limit $\lim\limits_{x\to\infty} \sqrt{x}\left(\sqrt{x+3} - \sqrt{x+1}\right)$ is:

A 0; B $\frac{1}{2}$; C 2; D 1.

3. In the parallelogram $ABCD$ we know that $AB = 1$, $AD = 2$ and $\widehat{B} = 60°$. Which of the following statements are true?

A $CD = 2$; B $BC = 2$; C $AC = \sqrt{3}$; D $BD = \sqrt{3}$.

4. Let A be a set with n elements. If the number of subsets of A having $(n-2)$ elements is 10, then

A $n \in (2, 6]$;
B $n \in (6, 10]$;
C $n \in (10, 14]$;
D there are no such values of n.

5. Let S be the set of real solutions of the equation

$$4^x - 2^x \cdot 5^{x+1} = 6 \cdot 25^x.$$

Which of the following statements are true?

A S has exactly two elements;
B S has exactly one element;
C $\dfrac{1+\log_2 3}{1+\log_2 5} \in S$;
D $\dfrac{1+\log_2 3}{1-\log_2 5} \in S$.

6. In the triangle ABC we have $E \in (AB)$, $EB = 2 \cdot EA$, $F \in (AC)$ and $FA = 3 \cdot FC$. Given that the points A, E and F have coordinates $A(1,3)$, $E(3,6)$ and $F(4,18)$, the coordinates of the centroid (center of mass) G of the triangle ABC are:

A $G\left(\frac{13}{3}, \frac{38}{3}\right)$; B $G\left(\frac{23}{9}, 10\right)$; C $G\left(\frac{47}{9}, \frac{70}{3}\right)$; D $G(7, 26)$.

7. In the triangle ABC the points $D(1,5)$, $E(-4,4)$ and $F(6,2)$ are the midpoints of the sides AB, BC and AC, respectively. The area of the triangle ABC is:

A 10; B 20; C 40; D 80.

8. The value of the integral $\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\,\mathrm{d}x$ is:

A $\frac{\pi}{4}$; B $\frac{\pi}{2}$; C 1; D π.

9. For the real number a, the function $f : \mathbb{R} \to \mathbb{R}$ is given by $f(x) = 1 + x + axe^{-x^2}$. Consider the points $A(0,1)$ and $B(-1,3)$. The value of a for which the line AB is tangent to the graph of f in the point A is:

A -5; B -3; C 0; D 3.

10. In the parallelogram $ABCD$ we have $A(-2,1)$, $B(2,3)$ and $C(5,3)$. The equation of the line BD is:

A $2x - y - 1 = 0$; B $x - 2y + 4 = 0$; C $2x + y - 1 = 0$; D $x + 2y + 4 = 0$.

11. Let $\alpha \in (\pi, 2\pi)$ with $\cos(\alpha) = -\frac{1}{4}$. Which of the following statements are true?

A $\sin(\alpha) = -\frac{\sqrt{15}}{4}$; B $\sin(2\alpha) = -\frac{\sqrt{15}}{8}$; C $\cos(2\alpha) = -\frac{7}{8}$; D $\mathrm{tg}(\alpha) = -\sqrt{15}$.

12. Let $\vec{i}$ and $\vec{j}$ be the versors of a cartesian system. If the vectors $\vec{u} = 2\vec{i} + (p-1)\vec{j}$ and $\vec{v} = 8\vec{i} - 3\vec{j}$ are parallel, then the parameter $p \in \mathbb{R}$ can be:

A $\frac{1}{4}$; B $\frac{7}{4}$; C $\frac{19}{3}$; D 6.

13. The integer numbers b_1, b_2, b_3, b_4, b_5 are in a geometric progression with ratio $q = 3$ and $S = b_1 + b_2 + b_3 + b_4 + b_5$. Which of the following statements are correct?

A S is divisible by 11.
B S is a perfect square if and only if b_1 is a perfect square.
C If b_1 is odd, then S is even.
D If b_1 is odd, then S is odd.

14. Let a be a real parameter and consider the system of equations:

$$\begin{cases} x + \ \ y - z &= a \\ x + 2y - z &= 0 \\ x + ay + z &= 1. \end{cases}$$

Which of the following statements are correct?

- A For every $a \in \mathbb{R}$ the determinant of the matrix of the system is non-zero.
- B There exists $a \in \mathbb{R}$ for which the system has at least two solutions.
- C If $a = 1$, then $x + y + z = 1$.
- D There exists $a \in \mathbb{R}$ such that $x + y + z = 0$.

15. In the permutation group S_4 consider the elements

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} \quad \text{and} \quad \tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}.$$

If $x \in S_4$ is a permutation such that $\sigma x = \tau$, then

- A x is not uniquely determined;
- B x is uniquely determined;
- C $x^2 = \sigma$;
- D x^2 is the identity permutation.

16. Let $a, b \in \mathbb{R}$ and let $f : \mathbb{R} \to \mathbb{R}$ the function defined by

$$f(x) = \begin{cases} \dfrac{\sin(ax)}{x}, & \text{if } x < 0 \\ \mathrm{e}^{bx} + 2\sin x, & \text{if } x \geq 0. \end{cases}$$

If f is differentiable on $\mathbb{R}$, then the value of the sum $a + b$ is:

- A 1;
- B 0;
- C -2;
- D -1.

17. Given a real number a, consider the function $f : \mathbb{R} \to \mathbb{R}$, defined by $f(x) = \dfrac{x^2 + ax}{\sqrt{x^2+1}}$. The set of values a for which f has a local extremum point situated at distance 1 from the Oy axis is:

- A $\{-3, 3\}$;
- B $\{-3\}$;
- C $\{3\}$;
- D the empty set.

18. Let $f : [-\pi, \pi] \to \mathbb{R}$ be the function defined by $f(x) = \int_{-\pi}^{x} t \sin t \, \mathrm{d}t$. Which of the following statements are true?

- A $x = 0$ is a local extremum point for f;
- B f is strictly increasing on $[-\pi, \pi]$;
- C f is strictly decreasing on $[-\pi, \pi]$;
- D $x = 0$ is an inflection point for f.

19. Let $ABCDEF$ be a regular hexagon with side length 1. Which of the following statements are true?

A $\overrightarrow{AB} \cdot \overrightarrow{BC} = -\frac{1}{2}$; B $\overrightarrow{AB} \cdot \overrightarrow{CD} = -\frac{1}{2}$; C $\overrightarrow{AB} \cdot \overrightarrow{DE} = -\frac{1}{2}$; D $\overrightarrow{AB} \cdot \overrightarrow{EF} = -\frac{1}{2}$.

20. In the square $ABCD$ we have $A(1,0)$ și $B(5,2)$. The equation of the line CD can be:

A $x - 2y - 11 = 0$; B $x - 2y - 1 = 0$; C $x - 2y + 9 = 0$; D $x + 2y - 1 = 0$.

21. For every matrix $X \in \mathcal{M}_3(\mathbb{R})$ we write $\operatorname{Tr}(X)$ for the sum of the elements on the main diagonal of the matrix X. If

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{pmatrix},$$

then the value of the expression $\det(A^2) - \operatorname{Tr}(A^2)$ is

A 21; B 22; C 23; D 24.

22. If x_1, x_2 and x_3 are the roots of the polynomial

$$f = X^3 + X^2 + 6X + 2$$

then the value of the expression $\frac{x_2 + x_3}{x_1} + \frac{x_1 + x_3}{x_2} + \frac{x_1 + x_2}{x_3}$ is equal to

A 1; B 0; C i; D $-i$.

23. Let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = \frac{\cos x}{1 + \mathrm{e}^x}$. The area of the set in the plane enclosed by the graph of f, the Ox axis and the lines of the equations $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ is:

A 0; B $\frac{1}{2}$; C 1; D 2.

24. The value of the limit $\lim\limits_{n \to \infty} \sqrt[n]{\frac{(n+1)(n+2)\cdots(n+n)}{n^n}}$ is:

A e; B $\frac{4}{\mathrm{e}}$; C $\frac{1}{\mathrm{e}}$; D $\frac{2}{\mathrm{e}}$.

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Correct Answers

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1. C
2. D
3. B, C
4. A
5. B, D
6. A
7. C
8. A
9. B
10. A
11. A, C
12. A
13. A, B, D
14. A, C
15. B, D
16. D
17. A
18. B, D
19. B, D
20. A, C
21. B
22. B
23. C
24. B