STUDIA UNIV. "BABEŞ–BOLYAI", MATHEMATICA, Volume **LII**, Number 1, March 2007

ON THE TRANSFORMATIONS OF N-LINEAR CONNECTIONS IN THE k-OSCULATOR BUNDLE

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Abstract. In the present paper we determine the transformations for the coefficients of an N-linear connection on Osc^2M, $Osc^3M, Osc^4M, ...,$ Osc^kM, $(k \geq 2, k \in N)$ by a transformation of nonlinear connections. We prove that the set $\mathcal{T}$ of the transformations of N-linear connections on $Osc^kM, (k \geq 2, k \in N)$, together with the composition of mappings isn't a group, but we give some groups which keep invariant a part of components of the local coefficients of an N-linear connection.

1. **Preliminaries**

The geometry of J_0^kM, $k \in N^*$, the k-jet bundle, discovered by Ch. Ehresmann [4], was largely investigated by many scholars: P. Liebermann [9], M. Crampin [3], A. Kawaguchi [6], I. Kolar [7], D. Krupka [8], M. de Léon [10], W. Sarlet [3], F. Cantrjin [3], W.M. Tulczyew [19], D. Grigore [5], R. Miron [4] et al. [12, 13, 14, 15, 16].

Generally, the geometries of higher order defined as the study of the category of bundles of jet $(J_0^kM\ ,\ \pi^k,\ M)$ are based on a direct approach of the properties of objects and morphisms in this category, without local coordinates.

But, many mathematical models from Lagrangian Mechanics, Theoretical Physics and Variational Calculus used multivariate Lagrangians of higher order acceleration, L$(x,\ \frac{dx}{dt}\ (t),\ ...\ ,\ \frac{1}{k!}\frac{d^kx}{dtk}\ (t)\ )$, (see E. Cartan, [2], for k=2, etc.).

Received by the editors: 22.11.2005.

2000 *Mathematics Subject Classification.* 53C05, 53C20, 53C60, 53B40, 58B20.

Key words and phrases. tangent bundle, metrical structure, almost symplectic structure, integrability, invariants, semi-symmetric metrical d-linear connection, transformations group.

From here one can see the reason of construction of the geometry of the total space of the bundle of higher order accelerations (or the oscuator bundle of higher order) in local coordinates.

Recently, this construction was achived by R. Miron and Gh. Atanasiu in their joint papers [13, 14].

Let M be a real C^∞ manifold with n-dimensions, and $\left(Osc^k M, \pi^k, M\right)$ $(k \geq 2, k \in \mathbb{N})$ its k-osculator bundle. The local coordinates on the $(k+1)\,n$-dimensional manifold $Osc^k M, (k \geq 2, k \in \mathbb{N})$ are denoted by $\left(x^i, y^{(1)i}, y^{(2)i}, ..., y^{(k)i}\right)$.

Let N be a nonlinear connection on $Osc^k M, (k \geq 2, k \in \mathbb{N})$ with the coefficients

$$\left(\underset{(1)}{N}{}^i_j,\ \underset{(2)}{N}{}^i_j, ...,\ \underset{(k)}{N}{}^i_j\right), (k \geq 2, k \in \mathbb{N}), \left(i, j = \overline{1,n}\right).$$

Hence, the tangent space of $Osc^k M, (k \geq 2, k \in \mathbb{N})$ in the point $u = \left(x, y^{(1)}, y^{(2)}, ..., y^{(k)}\right) \in Osc^k M, (k \geq 2, k \in \mathbb{N})$ is given by the direct sum of the vector spaces:

$$T_u Osc^k M, (k \geq 2, k \in \mathbb{N}) = N_0(u) \oplus N_1(u) \oplus ... \oplus N_{k-1}(u) \oplus V_k(u),$$
$$\forall u \in Osc^k M, (k \geq 2, k \in \mathbb{N}). \quad (1.1)$$

An adapted basis to (1.1) is given by:

$$\left\{\frac{\delta}{\delta x^i}, \frac{\delta}{\delta y^{(1)i}}, \frac{\delta}{\delta y^{(2)i}}, ..., \frac{\delta}{\delta y^{(k)i}}\right\}(k \geq 2, k \in \mathbb{N}), \left(i = \overline{1,n}\right), \quad (1.2)$$

where

$$\begin{cases} \frac{\delta}{\delta x^i} = \frac{\partial}{\partial x^i} - \underset{(1)}{N}{}^j_i \frac{\partial}{\partial y^{(1)j}} - \underset{(2)}{N}{}^j_i \frac{\partial}{\partial y^{(2)j}} - ... - \underset{(k)}{N}{}^j_i \frac{\partial}{\partial y^{(k)j}}, \\ \frac{\delta}{\delta y^{(1)i}} = \frac{\partial}{\partial y^{(1)i}} - \underset{(1)}{N}{}^j_i \frac{\partial}{\partial y^{(2)j}} - \underset{(2)}{N}{}^j_i \frac{\partial}{\partial y^{(3)j}} - ... - \underset{(k-1)}{N}{}^j_i \frac{\partial}{\partial y^{(k)j}}, \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ \frac{\delta}{\delta y^{(k-1)i}} = \frac{\partial}{\partial y^{(k-1)i}} - \underset{(1)}{N}{}^j_i \frac{\partial}{\partial y^{(k)j}}, \\ \frac{\delta}{\delta y^{(k)i}} = \frac{\partial}{\partial y^{(k)i}}, (k \geq 2, k \in \mathbb{N}), \left(i, j = \overline{1,n}\right). \end{cases} \quad (1.3)$$

Let us consider the dual basis of (1.2) :

$$\left\{dx^i, \delta y^{(1)i}, \delta y^{(2)i}, ..., \delta y^{(k)i}\right\}, (k \geq 2, k \in \mathbb{N}), \left(i = \overline{1,n}\right), \tag{1.4}$$

where:

$$\begin{cases} \delta x^i = dx^i, \\ \delta y^{(1)i} = dy^{(1)i} + \underset{(1)}{M}{}^i_j dx^j, \\ \delta y^{(2)i} = dy^{(2)i} + \underset{(1)}{M}{}^i_j dy^{(1)j} + \underset{(2)}{M}{}^i_j dx^j, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \delta y^{(k)i} = dy^{(k)i} + \underset{(1)}{M}{}^i_j dy^{(k-1)j} + ... + \underset{(k-1)}{M}{}^i_j dy^{(1)j} + \underset{(k)}{M}{}^i_j dx^j, \end{cases} \tag{1.5}$$

where

$$\begin{cases} \underset{(1)}{M}{}^i_j = \underset{(1)}{N}{}^i_j, \underset{(2)}{M}{}^i_j = \underset{(2)}{N}{}^i_j + \underset{(1)}{N}{}^i_m \underset{(1)}{M}{}^m_j, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \underset{(k)}{M}{}^i_j = \underset{(k)}{N}{}^i_j + \underset{(k-1)}{N}{}^i_m \underset{(1)}{M}{}^m_j + ... + \underset{(2)}{N}{}^i_m \underset{(k-2)}{M}{}^m_j + \underset{(1)}{N}{}^i_m \underset{(k-1)}{M}{}^m_j. \end{cases} \tag{1.6}$$

Let D be an N-linear connection on $Osc^k M, (k \geq 2, k \in \mathbb{N})$ with the local coefficients in the adapted basis: $D\Gamma(N) = \left(L^i_{jk}, \underset{(\alpha)}{C}{}^i_{jk}\right), (k \geq 2, k \in \mathbb{N})$.

The terminology and notations are usually retained, which are essentially based on the R. Miron's book: [12].

2. The set of the transformations of N-linear connections

Let $\overline{N}$ be another nonlinear connection on $Osc^k M, (k \geq 2, k \in \mathbb{N})$ with the coefficients $\left(\underset{(1)}{\overline{N}}{}^i_j, \underset{(2)}{\overline{N}}{}^i_j, ..., \underset{(k)}{\overline{N}}{}^i_j\right)$.Then there exists the uniquely determined tensor fields $\underset{(\alpha)}{A}{}^i_j \in \tau^1_1\left(Osc^k M, (k \geq 2, k \in \mathbb{N})\right), \left(\alpha = \overline{1,k}\right)$ on $Osc^k M, (k \geq 2, k \in \mathbb{N})$ such that:

$$\underset{(\alpha)}{\overline{N}}{}^i_j = \underset{(\alpha)}{N}{}^i_j - \underset{(\alpha)}{A}{}^i_j, \left(\alpha = \overline{1,k}\right)(k \geq 2, k \in \mathbb{N}). \tag{2.1}$$

Conversely, if $\underset{(\alpha)}{N}{}^i_j$ and $\underset{(\alpha)}{A}{}^i_j$ $\left(\alpha = \overline{1,k}\right)(k \geq 2, k \in \mathbb{N})$ are given, then $\underset{(\alpha)}{\overline{N}}{}^i_j$, $\left(\alpha = \overline{1,k}\right)(k \geq 2, k \in \mathbb{N})$, given by (2.1) is a nonlinear connection.

Let us suppose that the mapping $N \to \overline{N}$ is given by (2.1)

Let $\overline{D}$ be an $\overline{N}$-linear connection on $Osc^k M, (k \geq 2, k \in \mathbb{N})$ with the local coefficients in the adapted basis: $D\overline{\Gamma}\left(\overline{N}\right) = \left(\overline{L}^i_{jk}, \underset{(\alpha)}{\overline{C}}{}^i_{jk}\right), (k \geq 2, k \in \mathbb{N})$.

According to [12] we have:

$$\begin{cases} D_{\frac{\delta}{\delta x^j}} \frac{\delta}{\delta x^i} = L^m_{ij} \frac{\delta}{\delta x^m}, D_{\frac{\delta}{\delta x^j}} \frac{\delta}{\delta y^{(\alpha)i}} = L^m_{ij} \frac{\delta}{\delta y^{(\alpha)m}}, \left(\alpha = \overline{1,k}\right) (k \geq 2, k \in \mathbb{N}), \\ D_{\frac{\delta}{\delta y^{(\beta)j}}} \frac{\delta}{\delta x^i} = \underset{(\beta)}{C}{}^m_{ij} \frac{\delta}{\delta x^m}, D_{\frac{\delta}{\delta y^{(\beta)j}}} \frac{\delta}{\delta y^{(\alpha)i}} = \underset{(\beta)}{C}{}^m_{ij} \frac{\delta}{\delta y^{(\alpha)m}}, \left(\alpha, \beta = \overline{1,k}\right) (k \geq 2, k \in \mathbb{N}), \end{cases} \tag{2.2}$$

The adapted basis corresponding to the nonlinear connection $\overline{N}$ is:

$$\begin{cases} \frac{\overline{\delta}}{\delta x^i} = \frac{\partial}{\partial x^i} - \underset{(1)}{\overline{N}}{}^j_i \frac{\partial}{\partial y^{(1)j}} - \underset{(2)}{\overline{N}}{}^j_i \frac{\partial}{\partial y^{(2)j}} - ... - \underset{(k)}{\overline{N}}{}^j_i \frac{\partial}{\partial y^{(k)j}}, \\ \frac{\delta}{\delta y^{(1)i}} = \frac{\partial}{\partial y^{(1)i}} - \underset{(1)}{\overline{N}}{}^j_i \frac{\partial}{\partial y^{(2)j}} - \underset{(2)}{\overline{N}}{}^j_i \frac{\partial}{\partial y^{(3)j}} - ... - \underset{(k-1)}{\overline{N}}{}^j_i \frac{\partial}{\partial y^{(k)j}}, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \frac{\overline{\delta}}{\delta y^{(k-1)i}} = \frac{\partial}{\partial y^{(k-1)i}} - \underset{(1)}{\overline{N}}{}^j_i \frac{\partial}{\partial y^{(k)j}}, \\ \frac{\delta}{\delta y^{(k)i}} = \frac{\partial}{\partial y^{(k)i}}. \end{cases} \tag{2.3}$$

It follows first of all that the transformations (2.1) preserve the coefficients $\underset{(k)}{C}{}^i_{jk}$. From $(1.3), (2.3)$ and (2.1) we obtain:

$$\begin{cases} \frac{\overline{\delta}}{\delta x^i} = \frac{\delta}{\delta x^i} + \underset{(1)}{A}{}^j_i \frac{\partial}{\partial y^{(1)j}} + \underset{(2)}{A}{}^j_i \frac{\partial}{\partial y^{(2)j}} + ... + \underset{(k)}{A}{}^j_i \frac{\partial}{\partial y^{(k)j}}, \\ \frac{\overline{\delta}}{\delta y^{(1)i}} = \frac{\delta}{\delta y^{(1)i}} + \underset{(1)}{A}{}^j_i \frac{\partial}{\partial y^{(2)j}} + \underset{(2)}{A}{}^j_i \frac{\partial}{\partial y^{(3)j}} + ... + \underset{(k-1)}{A}{}^j_i \frac{\partial}{\partial y^{(k)j}}, \\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\ \frac{\overline{\delta}}{\delta y^{(k-1)i}} = \frac{\delta}{\delta y^{(k-1)i}} + \underset{(1)}{A}{}^j_i \frac{\partial}{\partial y^{(k)j}}, \\ \frac{\overline{\delta}}{\delta y^{(k)i}} = \frac{\delta}{\delta y^{(k)i}}. \end{cases} \tag{2.4}$$

Using $(2.2), (2.4)$and (1.3) we have:

$$D_{\frac{\overline{\delta}}{\delta x^j}} \frac{\overline{\delta}}{\delta y^{(k)i}} = \overline{L}^m_{ij} \frac{\overline{\delta}}{\delta y^{(k)m}} = \overline{L}^m_{ij} \frac{\delta}{\delta y^{(k)m}},$$

$$D_{\frac{\overline{\delta}}{\delta x^j}} \frac{\overline{\delta}}{\delta y^{(k)i}} = D_{\left(\frac{\delta}{\delta x^j} + \underset{(1)}{A}{}^l_j \frac{\partial}{\partial y^{(1)l}} + \underset{(2)}{A}{}^l_j \frac{\partial}{\partial y^{(2)l}} + ... + \underset{(k)}{A}{}^l_j \frac{\partial}{\partial y^{(k)l}}\right)} \frac{\delta}{\delta y^{(k)i}} =$$

$$= D_{\frac{\delta}{\delta x^j}} \frac{\delta}{\delta y^{(k)i}} + \underset{(1)_j}{A}{}^{l} D_{\frac{\partial}{\partial y^{(1)l}}} \frac{\delta}{\delta y^{(k)i}} + \underset{(2)_j}{A}{}^{l} D_{\frac{\partial}{\partial y^{(2)l}}} \frac{\delta}{\delta y^{(k)i}} + \dots + \underset{(k)_j}{A}{}^{l} D_{\frac{\partial}{\partial y^{(k)l}}} \frac{\delta}{\delta y^{(k)i}} =$$

$$= L^m_{ij} \frac{\delta}{\delta y^{(k)m}} + \underset{(1)_j}{A}{}^{l} D_{\frac{\delta}{\delta y^{(1)l}} + \underset{(1)_l}{N}{}^{r} \frac{\partial}{\partial y^{(2)r}} + \underset{(2)_l}{N}{}^{r} \frac{\partial}{\partial y^{(3)r}} + \dots + \underset{(k-1)_l}{N}{}^{r} \frac{\partial}{\partial y^{(k)r}}} \frac{\delta}{\delta y^{(k)i}} +$$

$$+ \underset{(2)_j}{A}{}^{l} D_{\frac{\delta}{\delta y^{(2)l}} + \underset{(1)_l}{N}{}^{r} \frac{\partial}{\partial y^{(3)r}} + \underset{(2)_l}{N}{}^{r} \frac{\partial}{\partial y^{(4)r}} + \dots + \underset{(k-2)_l}{N}{}^{r} \frac{\partial}{\partial y^{(k)r}}} \frac{\delta}{\delta y^{(k)i}} + \dots + \underset{(k)_j}{A}{}^{l} D_{\frac{\delta}{\delta y^{(k)l}}} \frac{\delta}{\delta y^{(k)i}} =$$

$$= L^m_{ij} \frac{\delta}{\delta y^{(k)m}} + \left(\underset{(1)_j}{A}{}^{l} D_{\frac{\delta}{\delta y^{(1)l}}} \frac{\delta}{\delta y^{(k)i}} + \underset{(1)_j}{A}{}^{l} \underset{(1)_l}{N}{}^{r} D_{\frac{\partial}{\partial y^{(2)r}}} \frac{\delta}{\delta y^{(k)i}} + \underset{(1)_j}{A}{}^{l} \underset{(2)_l}{N}{}^{r} D_{\frac{\partial}{\partial y^{(3)r}}} \frac{\delta}{\delta y^{(k)i}} + \dots \right.$$

$$\left. + \underset{(1)_j}{A}{}^{l} \underset{(k-1)_l}{N}{}^{r} D_{\frac{\delta}{\delta y^{(k)r}}} \frac{\delta}{\delta y^{(k)i}} \right) + \left(\underset{(2)_j}{A}{}^{l} D_{\frac{\delta}{\delta y^{(2)l}}} \frac{\delta}{\delta y^{(k)i}} + \underset{(2)_j}{A}{}^{l} \underset{(1)_l}{N}{}^{r} D_{\frac{\partial}{\partial y^{(3)r}}} \frac{\delta}{\delta y^{(k)i}} + \right.$$

$$\left. + \underset{(2)_j}{A}{}^{l} \underset{(2)_l}{N}{}^{r} D_{\frac{\partial}{\partial y^{(4)r}}} \frac{\delta}{\delta y^{(k)i}} + \dots + \underset{(2)_j}{A}{}^{l} \underset{(k-2)_l}{N}{}^{r} D_{\frac{\delta}{\delta y^{(k)r}}} \frac{\delta}{\delta y^{(k)i}} \right) + \dots + \underset{(k)_j}{A}{}^{l} \underset{(k)_{il}}{C}{}^{m} \frac{\delta}{\delta y^{(k)m}} =$$

$$= L^m_{ij} \frac{\delta}{\delta y^{(k)m}} + \underset{(1)_j}{A}{}^{l} \underset{(1)_{il}}{C}{}^{m} \frac{\delta}{\delta y^{(k)m}} + \underset{(2)_j}{A}{}^{l} \underset{(2)_{il}}{C}{}^{m} \frac{\delta}{\delta y^{(k)m}} + \dots + \underset{(k)_j}{A}{}^{l} \underset{(k)_{il}}{C}{}^{m} \frac{\delta}{\delta y^{(k)m}} +$$

$$+ \underset{(1)_j}{A}{}^{l} \underset{(1)_l}{N}{}^{r} D_{\left(\frac{\delta}{\delta y^{(2)r}} + \underset{(1)_r}{N}{}^{s} \frac{\partial}{\partial y^{(3)s}} + \underset{(2)_r}{N}{}^{s} \frac{\partial}{\partial y^{(4)s}} + \dots + \underset{(k-2)_r}{N}{}^{s} \frac{\partial}{\partial y^{(k)s}} \right)} \frac{\delta}{\delta y^{(k)i}} +$$

$$+ \underset{(1)_j}{A}{}^{l} \underset{(2)_l}{N}{}^{r} D_{\left(\frac{\delta}{\delta y^{(3)r}} + \underset{(1)_r}{N}{}^{s} \frac{\partial}{\partial y^{(4)s}} + \underset{(2)_r}{N}{}^{s} \frac{\partial}{\partial y^{(5)s}} + \dots + \underset{(k-3)_r}{N}{}^{s} \frac{\partial}{\partial y^{(k)s}} \right)} \frac{\delta}{\delta y^{(k)i}} + \dots +$$

$$+ \underset{(1)_j}{A}{}^{l} \underset{(k-1)_l}{N}{}^{r} \underset{(k)_{ir}}{C}{}^{m} \frac{\delta}{\delta y^{(k)m}} + \dots$$

$$D_{\frac{\bar{\delta}}{\delta y^{(1)j}}} \frac{\bar{\delta}}{\delta y^{(k)i}} = \underset{(1)_{ij}}{\overline{C}}{}^{m} \frac{\delta}{\delta y^{(k)m}}, D_{\frac{\bar{\delta}}{\delta y^{(1)j}}} \frac{\bar{\delta}}{\delta y^{(k)i}} = D_{\left(\frac{\delta}{\delta y^{(1)j}} + \underset{(1)_j}{A}{}^{l} \frac{\partial}{\partial y^{(2)l}} + \dots + \underset{(k-1)_j}{A}{}^{l} \frac{\partial}{\partial y^{(k)l}} \right)} \frac{\delta}{\delta y^{(k)i}}$$

$$= \underset{(1)_{ij}}{C}{}^{m} \frac{\delta}{\delta y^{(k)m}} + \underset{(1)_j}{A}{}^{l} D_{\frac{\partial}{\partial y^{(2)l}}} \frac{\delta}{\delta y^{(k)i}} + \dots + \underset{(k-2)_j}{A}{}^{l} D_{\frac{\partial}{\partial y^{(k-1)l}}} \frac{\delta}{\delta y^{(k)i}} + \underset{(k-1)_j}{A}{}^{l} \underset{(k)_{il}}{C}{}^{m} \frac{\delta}{\delta y^{(k)m}}.$$

....

$$D_{\frac{\bar{\delta}}{\delta y^{(k-1)j}}} \frac{\bar{\delta}}{\delta y^{(k)i}} = \underset{(k-1)_{ij}}{\overline{C}}{}^{m} \frac{\delta}{\delta y^{(k)m}}, D_{\frac{\bar{\delta}}{\delta y^{(k-1)j}}} \frac{\bar{\delta}}{\delta y^{(k)i}} = D_{\left(\frac{\delta}{\delta y^{(k-1)j}} + \underset{(1)_j}{A}{}^{l} \frac{\partial}{\partial y^{(k)l}} \right)} \frac{\delta}{\delta y^{(k)i}} =$$

$$\underset{(k-1)_{ij}}{C}{}^{m} \frac{\delta}{\delta y^{(k)m}} + \underset{(1)_j}{A}{}^{l} \underset{(k)_{il}}{C}{}^{m} \frac{\delta}{\delta y^{(k)m}}.$$

Therefore the change we are looking for is:

$$
\begin{cases}
\overline{L}^{m}_{ij} = L^{m}_{ij} + \underset{(1)}{A}{}^{l}_{j}\Big[\underset{(1)}{C}{}^{m}_{il} + \underset{(1)}{N}{}^{r}_{l}\underset{(2)}{C}{}^{m}_{ir} + ... + \underset{(k-1)}{N}{}^{r}_{l}\underset{(k)}{C}{}^{m}_{ir} + \underset{(1)}{N}{}^{r}_{l}\underset{(1)}{N}{}^{s}_{r}\underset{(3)}{C}{}^{m}_{is} + \\
... + \left(\underset{(1)}{N}{}^{r}_{l}\underset{(k-2)}{N}{}^{s}_{r} + \underset{(2)}{N}{}^{r}_{l}\underset{(k-3)}{N}{}^{s}_{r} + ... + \underset{(k-2)}{N}{}^{r}_{l}\underset{(2)}{N}{}^{s}_{r} + \underset{(k-1)}{N}{}^{r}_{l}\underset{(1)}{N}{}^{s}_{r}\right)\underset{(k)}{C}{}^{m}_{is} + \\
... + \underbrace{\underset{(1)}{N}{}^{r}_{l}\underset{(1)}{N}{}^{s}_{r}...\underset{(1)}{N}}_{(k-1)}\underset{k}{C}\Big] + \underset{(2)}{A}{}^{l}_{j}\Big[\underset{(2)}{C}{}^{m}_{il} + \underset{(1)}{N}{}^{r}_{l}\underset{(3)}{C}{}^{m}_{ir} + ... + \\
+ \underset{(k-2)}{N}{}^{r}_{l}\underset{(k)}{C}{}^{m}_{ir} + ... + \underbrace{\underset{(1)}{N}{}^{r}_{l}\cdot ... \cdot\underset{(1)}{N}}_{(k-2)}\underset{(k)}{C}\Big] + ... + \\
+ \underset{(k-1)}{A}{}^{l}_{j}\Big[\underset{(k-1)}{C}{}^{m}_{il} + \underset{(1)}{N}{}^{r}_{l}\underset{(k)}{C}{}^{m}_{ir}\Big] + \underset{(k)}{A}{}^{l}_{j}\underset{(k)}{C}{}^{m}_{il}, \left(k \geq 2, k \in \mathbb{N}\right), \\
\underset{(1)}{\overline{C}}{}^{m}_{ij} = \underset{(1)}{C}{}^{m}_{ij} + \underset{(1)}{A}{}^{l}_{j}\Big[\underset{(2)}{C}{}^{m}_{il} + \underset{(1)}{N}{}^{r}_{l}\underset{(3)}{C}{}^{m}_{ir} + ... + \underset{(k-2)}{N}{}^{r}_{l}\underset{(k)}{C}{}^{m}_{ir} + ... + \\
+ \underbrace{\underset{(1)}{N}{}^{r}_{l}\cdot ... \cdot\underset{(1)}{N}}_{(k-2)}\underset{(k)}{C}\Big] + ... + \underset{(k-2)}{A}{}^{l}_{j}\Big[\underset{(k-1)}{C}{}^{m}_{il} + \underset{(1)}{N}{}^{r}_{l}\underset{(k)}{C}{}^{m}_{ir}\Big] + \underset{(k-1)}{A}{}^{l}_{j}\underset{(k)}{C}{}^{m}_{il}, \left(k \geq 2, k \in \mathbb{N}\right), \\
.... \\
\underset{(k-1)}{\overline{C}}{}^{m}_{ij} = \underset{(k-1)}{C}{}^{m}_{ij} + \underset{(1)}{A}{}^{l}_{j}\underset{(k)}{C}{}^{m}_{il}, \left(k \geq 2, k \in \mathbb{N}\right), \\
\underset{(k)}{\overline{C}}{}^{m}_{ij} = \underset{(k)}{C}{}^{m}_{ij}, \left(k \geq 2, k \in \mathbb{N}\right).
\end{cases}
\tag{2.5}
$$

So, we have proved:

Proposition 1. *The transformation* (2.1) *of nonlinear connections imply the transformations* (2.5)*for the coefficients*

$$D\Gamma\left(N\right) = \left(L^{i}_{jk}, \underset{(\alpha)}{C}{}^{i}_{jk}\right), \left(\alpha = \overline{1, k}\right), \left(k \geq 2, k \in \mathbb{N}\right).$$

Particular cases:

1. If we take $k = 2$ in (2.5) then we obtain a result given in [17]:

$$\begin{cases} \overline{L}^m_{ij} = L^m_{ij} + \underset{(1)_j}{A^{l}}\left(\underset{(1)_{il}}{C^{m}} + \underset{(1)_l}{N^{r}}\underset{(2)_{ir}}{C^{m}}\right) + \underset{(2)_j}{A^{l}}\underset{(2)_{il}}{C^{m}}, \\ \underset{(1)_{ij}}{\overline{C}^{m}} = \underset{(1)_{ij}}{C^{m}} + \underset{(1)_j}{A^{l}}\underset{(2)_{il}}{C^{m}}, \\ \underset{(2)_{ij}}{\overline{C}^{m}} = \underset{(2)_{ij}}{C^{m}}. \end{cases} \tag{2.6}$$

2.If we take $k = 3$ in (2.5), then we obtain the transformations for the coefficients of an N-linear connection on Osc^3M by a transformation of nonlinear connections, result given in [18]:

$$\begin{cases} \overline{L}^m_{ij} = L^m_{ij} + \underset{(1)_j}{A^{l}}\left(\underset{(1)_{il}}{C^{m}} + \underset{(1)_l}{N^{r}}\underset{(2)_{ir}}{C^{m}} + \underset{(1)_l}{N^{r}}\underset{(1)_r}{N^{s}}\underset{(3)_{is}}{C^{m}} + \underset{(2)_l}{N^{r}}\right)\underset{(3)_{ir}}{C^{m}} + \\ + \underset{(2)_j}{A^{l}}\left(\underset{(2)_{il}}{C^{m}} + \underset{(1)_l}{N^{r}}\underset{(3)_{ir}}{C^{m}}\right) + \underset{(3)_j}{A^{l}}\underset{(3)_{il}}{C^{m}}, \\ \underset{(1)_{ij}}{\overline{C}^{m}} = \underset{(1)_{ij}}{C^{m}} + \underset{(1)_j}{A^{l}}\left(\underset{(2)_{il}}{C^{m}} + \underset{(1)_l}{N^{r}}\underset{(3)_{ir}}{C^{m}}\right) + \underset{(2)_j}{A^{l}}\underset{(3)_{il}}{C^{m}}, \\ \underset{(2)_{ij}}{\overline{C}^{m}} = \underset{(2)_{ij}}{C^{m}} + \underset{(1)_j}{A^{l}}\underset{(3)_{il}}{C^{m}}, \\ \underset{(3)_{ij}}{\overline{C}^{m}} = \underset{(3)_{ij}}{C^{m}}. \end{cases} \tag{2.7}$$

3. If we consider $k = 4$ in (2.5), then we obtain the transformations for the coefficients of an N-linear connection on Osc^4M by a transformation of nonlinear connections.

$$
\begin{cases}
\overline{L}^{m}_{ij} = L^{m}_{ij} + \underset{(1)}{A}{}^{l}_{j}\left(\underset{(1)}{C}{}^{m}_{il} + \underset{(1)}{N}{}^{r}_{l}\underset{(2)}{C}{}^{m}_{ir} + \underset{(2)}{N}{}^{r}_{l}\underset{(3)}{C}{}^{m}_{ir} + \underset{(3)}{N}{}^{r}_{l}\underset{(4)}{C}{}^{m}_{ir} + \right. \\
+\underset{(1)}{N}{}^{r}_{l}\underset{(1)}{N}{}^{s}_{r}\underset{(3)}{C}{}^{m}_{is} + \underset{(1)}{N}{}^{r}_{l}\underset{(2)}{N}{}^{s}_{r}\underset{(4)}{C}{}^{m}_{is} + \underset{(2)}{N}{}^{r}_{l}\underset{(1)}{N}{}^{s}_{r}\underset{(4)}{C}{}^{m}_{is} + \\
\left. + \underset{(1)}{N}{}^{r}_{l}\underset{(1)}{N}{}^{s}_{r}\underset{(1)}{N}{}^{t}_{s}\underset{(4)}{C}{}^{m}_{it}\right) + \underset{(2)}{A}{}^{l}_{j}\left(\underset{(2)}{C}{}^{m}_{il} + \underset{(3)}{C}{}^{m}_{ir} + \underset{(2)}{N}{}^{r}_{l}\underset{(4)}{C}{}^{m}_{ir} + \right. \\
\left. + \underset{(1)}{N}{}^{r}_{l}\underset{(1)}{N}{}^{s}_{r}\underset{(4)}{C}{}^{m}_{is}\right) + \underset{(3)}{A}{}^{l}_{j}\left(\underset{(3)}{C}{}^{m}_{il} + \underset{(1)}{N}{}^{r}_{l}\underset{(4)}{C}{}^{m}_{ir}\right) + \underset{(4)}{A}{}^{l}_{j}\underset{(4)}{C}{}^{m}_{il}, \\
\underset{(1)}{\overline{C}}{}^{m}_{ij} = \underset{(1)}{C}{}^{m}_{ij} + \underset{(1)}{A}{}^{l}_{j}\left(\underset{(2)}{C}{}^{m}_{il} + \underset{(1)}{N}{}^{r}_{l}\underset{(3)}{C}{}^{m}_{ir} + \underset{(2)}{N}{}^{r}_{l}\underset{(4)}{C}{}^{m}_{ir} + \underset{(1)}{N}{}^{r}_{l}\underset{(1)}{N}{}^{s}_{r}\underset{(4)}{C}{}^{m}_{is}\right) + \\
+\underset{(2)}{A}{}^{l}_{j}\left(\underset{(3)}{C}{}^{m}_{il} + \underset{(1)}{N}{}^{r}_{l}\underset{(4)}{C}{}^{m}_{ir}\right) + \underset{(3)}{A}{}^{l}_{j}\underset{(4)}{C}{}^{m}_{il}, \\
\underset{(2)}{\overline{C}}{}^{m}_{ij} = \underset{(2)}{C}{}^{m}_{ij} + \underset{(1)}{A}{}^{l}_{j}\left(\underset{(3)}{C}{}^{m}_{il} + \underset{(1)}{N}{}^{r}_{l}\underset{(4)}{C}{}^{m}_{ir}\right) + \underset{(2)}{A}{}^{l}_{j}\underset{(4)}{C}{}^{m}_{il}, \\
\underset{(3)}{\overline{C}}{}^{m}_{ij} = \underset{(3)}{C}{}^{m}_{ij} + \underset{(1)}{A}{}^{l}_{j}\underset{(4)}{C}{}^{m}_{il}, \\
\underset{(4)}{\overline{C}}{}^{m}_{ij} = \underset{(4)}{C}{}^{m}_{ij}.
\end{cases}
\tag{2.8}
$$

etc.

Now, we can prove:

Theorem 1. *Let* N *and* $\overline{N}$ *be two nonlinear connections on* $Osc^k M$, $(k \geq 2$, $k \in \mathbb{N})$ *with coefficients*

$$\left(\underset{(1)}{N}{}^{i}_{j}, \underset{(2)}{N}{}^{i}_{j}, ..., \underset{(k)}{N}{}^{i}_{j}\right), \ \left(\underset{(1)}{\overline{N}}{}^{i}_{j}, \underset{(2)}{\overline{N}}{}^{i}_{j}, ..., \underset{(k)}{\overline{N}}{}^{i}_{j}\right), \ \left(\alpha = \overline{1,k}\right), \ (k \geq 2, k \in \mathbb{N})$$

respectively. If

$$D\Gamma(N) = \left(L^{m}_{ij}, \underset{(\alpha)}{C}{}^{m}_{ij}\right)$$

and

$$D\overline{\Gamma}\left(\overline{N}\right) = \left(\overline{L}^{m}_{ij}, \underset{(\alpha)}{\overline{C}}{}^{m}_{ij}\right), \left(\alpha = \overline{1,k}\right), (k \geq 2, k \in \mathbb{N})$$

are the local coefficients of two N-, respectively $\overline{N}-$*linear connections,* D, *respectively* $\overline{D}$ *on the differentiable manifold* $Osc^k M, (k \geq 2, k \in \mathbb{N})$, *then there exists only one system of tensor fields* $\left(\underset{(1)}{A}{}^{i}_{j}, \underset{(2)}{A}{}^{i}_{j}, ..., \underset{(k)}{A}{}^{i}_{j}, B^{m}_{ij}, \underset{(1)}{D}{}^{m}_{ij}, \underset{(2)}{D}{}^{m}_{ij}, ..., \underset{(k)}{D}{}^{m}_{ij}\right)$ *such that:*

$$\left\{\begin{array}{l}
\underset{(\alpha)_j}{\overline{N}}{}^{i} = \underset{(\alpha)_j}{N}{}^{i} - \underset{(\alpha)_j}{A}{}^{i}, \left(\alpha=\overline{1,k}\right), \left(k\geq 2, k\in\mathbb{N}\right), \\
\overline{L}^{m}_{ij} = L^{m}_{ij} + \underset{(1)_j}{A}{}^{l} \Big[\underset{(1)_{il}}{C}{}^{m} + \underset{(1)_l}{N}{}^{r}\underset{(2)_{ir}}{C}{}^{m} + ... + \underset{(k-1)_l}{N}{}^{r}\underset{(k)_{ir}}{C}{}^{m} + \underset{(1)_l}{N}{}^{r}\underset{(1)_r}{N}{}^{s}\underset{(3)_{is}}{C}{}^{m} + \\
... + \left(\underset{(1)_l}{N}{}^{r}\underset{(k-2)_r}{N}{}^{s} + \underset{(2)_l}{N}{}^{r}\underset{(k-3)_r}{N}{}^{s} + ... + \underset{(k-2)_l}{N}{}^{r}\underset{(2)_r}{N}{}^{s} + \underset{(k-1)_l}{N}{}^{r}\underset{(1)_r}{N}{}^{s} \right) \underset{(k)_{is}}{C}{}^{m} + \\
... + \underbrace{\underset{(1)_l}{N}{}^{r}\underset{(1)_r}{N}{}^{s} ... \underset{(1)}{N}}_{(k-1)}\underset{(k)}{C} \Big] + \underset{(2)_j}{A}{}^{l} \Big[\underset{(2)_{il}}{C}{}^{m} + \underset{(1)_l}{N}{}^{r}\underset{(3)_{ir}}{C}{}^{m} + ... + \\
+ \underset{(k-2)_l}{N}{}^{r}\underset{(k)_{ir}}{C}{}^{m} + ... + \underbrace{\underset{(1)_l}{N}{}^{r} \cdot ... \cdot \underset{(1)}{N}}_{(k-2)}\underset{(k)}{C} \Big] + ... + \\
+ \underset{(k-1)_j}{A}{}^{l} \Big[\underset{(k-1)_{il}}{C}{}^{m} + \underset{(1)_l}{N}{}^{r}\underset{(k)_{ir}}{C}{}^{m} \Big] + \underset{(k)_j}{A}{}^{l}\underset{(k)_{il}}{C}{}^{m} - B^{m}_{ij}, \left(k\geq 2, k\in N\right), \\
\underset{(1)_{ij}}{\overline{C}}{}^{m} = \underset{(1)_{ij}}{C}{}^{m} + \underset{(1)_j}{A}{}^{l} \Big[\underset{(2)_{il}}{C}{}^{m} + \underset{(1)_l}{N}{}^{r}\underset{(3)_{ir}}{C}{}^{m} + ... + \underset{(k-2)_l}{N}{}^{r}\underset{(k)_{ir}}{C}{}^{m} + ... + + \underbrace{\underset{(1)_l}{N}{}^{r} \cdot ... \cdot \underset{(1)}{N}}_{(k-2)}\underset{(k)}{C} \Big] \\
+ \cdots + \underset{(k-2)_j}{A}{}^{l} \Big[\underset{(k-1)_{il}}{C}{}^{m} + \underset{(1)_l}{N}{}^{r}\underset{(k)_{ir}}{C}{}^{m} \Big] + \underset{(k-1)_j}{A}{}^{l}\underset{(k)_{il}}{C}{}^{m} - \underset{(1)_{ij}}{D}{}^{m}, \left(k\geq 2, k\in\mathbb{N}\right), \\
\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \\
\underset{(k-1)_{ij}}{\overline{C}}{}^{m} = \underset{(k-1)_{ij}}{C}{}^{m} + \underset{(1)_j}{A}{}^{l}\underset{(k)_{il}}{C}{}^{m} - \underset{(k-1)_{ij}}{D}{}^{m}, \left(k\geq 2, k\in\mathbb{N}\right), \\
\underset{(k)_{ij}}{\overline{C}}{}^{m} = \underset{(k)_{ij}}{C}{}^{m} - \underset{(k)_{ij}}{D}{}^{m}, \left(k\geq 2, k\in\mathbb{N}\right).
\end{array}\right. \tag{2.9}$$

Proof. The first equality (2.9) determines uniquely the tensor fields $\underset{(\alpha)_j}{A}{}^{i}, \left(\alpha=\overline{1,k}\right), \left(k\geq 2, k\in\mathbb{N}\right)$. Since $\underset{(\alpha)_{ij}}{C}{}^{m}, \left(\alpha=\overline{1,k}\right), \left(k\geq 2, k\in\mathbb{N}\right)$ are tensor fields, the second equation (2.9) determines uniquely the tensor field B^{m}_{ij}. Similarly the third, the fourth,...and the last equation (2.9)determine the tensor fields $\underset{(1)_{ij}}{D}{}^{m}, \underset{(2)_{ij}}{D}{}^{m}, ...$ and $\underset{(k)_{ij}}{D}{}^{m}, (k\geq 2, k\in\mathbb{N})$ respectively. □

We have immediately:

Theorem 2. *If* $D\Gamma(N) = \left(L^m_{ij}, \underset{(\alpha)}{C}{}^m_{ij}\right), \left(\alpha = \overline{1,k}\right), (k \geq 2, k \in \mathbb{N})$ *are the local coefficients on an N-linear connection D on* Osc^kM $(k \geq 2, k \in \mathbb{N})$ *and*

$$\left(\underset{(1)}{A}{}^i_j, \underset{(2)}{A}{}^i_j, ..., \underset{(k)}{A}{}^i_j, B^m_{ij}, \underset{(1)}{D}{}^m_{ij}, \underset{(2)}{D}{}^m_{ij}, ..., \underset{(k)}{D}{}^m_{ij}\right), (k \geq 2, k \in \mathbb{N})$$

is a system of tensor fields on Osc^kM $(k \geq 2, k \in \mathbb{N})$ *then*

$$D\overline{\Gamma}\left(\overline{N}\right) = \left(\overline{L}^m_{ij}, \underset{(\alpha)}{\overline{C}}{}^m_{ij}\right), \left(\alpha = \overline{1,k}\right), (k \geq 2, k \in \mathbb{N})$$

given by (2.9) *are the local coefficients of an* $\overline{N}-$*linear connection,* $\overline{D}$, *on* Osc^kM; $(k \geq 2, k \in \mathbb{N})$.

The system of tensor fields

$$\left(\underset{(1)}{A}{}^i_j, \underset{(2)}{A}{}^i_j, ..., \underset{(k)}{A}{}^i_j, B^m_{ij}, \underset{(1)}{D}{}^m_{ij}, \underset{(2)}{D}{}^m_{ij}, ..., \underset{(k)}{D}{}^m_{ij}\right), \ (k \geq 2, k \in \mathbb{N})$$

is called the difference tensor fields of $D\Gamma(N)$ to $D\overline{\Gamma}\left(\overline{N}\right)$ and the mapping $D\Gamma(N) \rightarrow D\overline{\Gamma}\left(\overline{N}\right)$ given by (2.9) is called a transformation of N-linear connection to $\overline{N}-$linear connection on Osc^kM $(k \geq 2, k \in \mathbb{N})$ and is noted by:

$$t\left(\underset{(1)}{A}{}^i_j, \underset{(2)}{A}{}^i_j, ..., \underset{(k)}{A}{}^i_j, B^m_{ij}, \underset{(1)}{D}{}^m_{ij}, \underset{(2)}{D}{}^m_{ij}, ..., \underset{(k)}{D}{}^m_{ij}\right), (k \geq 2, k \in \mathbb{N}).$$

Theorem 3.*The set* $\mathcal{T}$ *of the transformations of N-linear connections to* $\overline{N}-$*linear connection on* Osc^kM $(k \geq 2, k \in \mathbb{N})$ *together with the composition of mappings ist't a group.*

Proof. Let

$$t\left(\underset{(1)}{A}{}^i_j, \underset{(2)}{A}{}^i_j, ..., \underset{(k)}{A}{}^i_j, B^m_{ij}, \underset{(1)}{D}{}^m_{ij}, \underset{(2)}{D}{}^m_{ij}, ..., \underset{(k)}{D}{}^m_{ij}\right) : D\Gamma(N) \rightarrow D\overline{\Gamma}\left(\overline{N}\right)$$

and

$$t\left(\underset{(1)}{\overline{A}}{}^i_j, \underset{(2)}{\overline{A}}{}^i_j, ..., \underset{(k)}{\overline{A}}{}^i_j, B^m_{ij}, \underset{(1)}{D}{}^m_{ij}, \underset{(2)}{D}{}^m_{ij}, ..., \underset{(k)}{D}{}^m_{ij}\right) : D\overline{\Gamma}\left(\overline{N}\right) \rightarrow D\overline{\overline{\Gamma}}\left(\overline{\overline{N}}\right), \ (k \geq 2, k \in \mathbb{N})$$

be two transformations from $\mathcal{T}$, given by (2.9).

From (2.9) we have:

$$\underset{(\alpha)}{\overline{\overline{N}}}{}^i_j = \underset{(\alpha)}{N}{}^i_j - \left(\underset{(\alpha)}{A}{}^i_j + \underset{(\alpha)}{\overline{A}}{}^i_j\right), \left(\alpha = \overline{1,k}\right), (k \geq 2, k \in \mathbb{N}).$$

We obtain for example:

$$\underset{(k-1)_{ij}}{\overline{\overline{C}}}{}^{m} = \underset{(k-1)_{ij}}{C}{}^{m} + \underset{(k)_{il}}{C}{}^{m}\left(\underset{(1)_{j}}{A}{}^{l} + \underset{(1)_{j}}{\overline{A}}{}^{l}\right)\left(\underset{(k)_{il}}{D}{}^{m}\underset{(1)_{j}}{\overline{A}}{}^{l} + \underset{(k-1)_{ij}}{D}{}^{m} + \underset{(k-1)_{ij}}{\overline{D}}{}^{m}\right),$$

So, $\underset{(k-1)_{ij}}{\overline{\overline{C}}}{}^{m}$ hasn't the form (2.9). Result that the composition of two transformations from $\mathcal{T}$, isn't a transformation from $\mathcal{T}$, so $\mathcal{T}$ together with the composition of mappings isn't a group. □

Remark 1. If we consider $\underset{(\alpha)_{j}}{A}{}^{i}, \left(\alpha = \overline{1,k}\right), (k \geq 2, k \in \mathbb{N})$ in (2.9) we obtain the set $\mathcal{T}_N$ of transformations of N-linear connections corresponding to the same nonlinear connection N:

$$\mathcal{T}_N = \left\{ t\left(\underbrace{0,0,...,0}_{(k)}, B_{ij}^{m}, \underset{(1)_{ij}}{D}{}^{m}, \underset{(2)_{ij}}{D}{}^{m}, ..., \underset{(k)_{ij}}{D}{}^{m}\right) \in \mathcal{T}\,(k \geq 2, k \in \mathbb{N}) \right\}.$$

We have:

Theorem 4. The set $\mathcal{T}_N$ of the transformations of N-linear connections to N-linear connections on $Osc^k M$ $(k \geq 2, k \in \mathbb{N})$ together with the composition of mappings ist't a group. This group $\mathcal{T}_N$ acts effectively and transitivelly on the set of N-linear connections.

Proof. Let $t\left(\underbrace{0,0,...,0}_{(k)}, B_{ij}^{m}, \underset{(1)_{ij}}{D}{}^{m}, \underset{(2)_{ij}}{D}{}^{m}, ..., \underset{(k)_{ij}}{D}{}^{m}\right) : D\Gamma(N) \to D\overline{\Gamma}(N), (k \geq 2, k \in \mathbb{N})$ be a transformation from $\mathcal{T}_N$ given by (2.10) : □

Proof.

$$\begin{cases} \underset{(\alpha)_{j}}{\overline{N}}{}^{i} = \underset{(\alpha)_{j}}{N}{}^{i}, \left(\alpha = \overline{1,k}\right), (k \geq 2, k \in \mathbb{N}), \\ \underset{(\alpha)_{ij}}{\overline{L}}{}^{m} = L_{ij}^{m} - B_{ij}^{m}, \\ \underset{(\alpha)_{ij}}{\overline{C}}{}^{m} = \underset{(\alpha)_{ij}}{C}{}^{m} - \underset{(\alpha)_{ij}}{N}{}^{m}, \left(\alpha = \overline{1,k}\right), (k \geq 2, k \in \mathbb{N}). \end{cases} \tag{2.10}$$

The composition of two transformations from $\mathcal{T}_N$ is a transformation from $\mathcal{T}_N$, given by:

$$t\left(\underbrace{0,0,...,0}_{(k)}, \overline{B}_{ij}^{m}, \underset{(1)_{ij}}{\overline{D}}{}^{m}, \underset{(2)_{ij}}{\overline{D}}{}^{m}, ..., \underset{(k)_{ij}}{\overline{D}}{}^{m}\right) \circ t\left(\underbrace{0,0,...,0}_{(k)}, B_{ij}^{m}, \underset{(1)_{ij}}{D}{}^{m}, \underset{(2)_{ij}}{D}{}^{m}, ..., \underset{(k)_{ij}}{D}{}^{m}\right) =$$

$$t\left(\underbrace{0,0,...,0}_{(k)},B^m_{ij}+\overline{B}^m_{ij},\underset{(1)_{ij}}{D}{}^m+\underset{(1)_{ij}}{\overline{D}}{}^m,\underset{(2)_{ij}}{D}{}^m+\underset{(2)_{ij}}{\overline{D}}{}^m,...,\underset{(k)_{ij}}{D}{}^m+\underset{(k)_{ij}}{\overline{D}}{}^m\right).$$

The inverse of a transformation from $\mathcal{T}_N$ is the transformation:

$$t\left(\underbrace{0,0,...,0}_{(k)},-B^m_{ij},-\underset{(1)_{ij}}{D}{}^m,-\underset{(2)_{ij}}{D}{}^m,...,-\underset{(k)_{ij}}{D}{}^m\right):D\Gamma(N)\to D\overline{\Gamma}(N).$$

The transformation (2.10) preserves all the N-linear connections D if $B^m_{ij}=\underset{(\alpha)_{ij}}{D}{}^m=0,\left(\alpha=\overline{1,k}\right),(k\geq 2,k\in\mathbb{N})$. Therefore $\mathcal{T}_N$ acts effectively on the set of N-linear connections. From the theorem 1. results that $\mathcal{T}_N$ acts transitively on this set. □

Let be:

$$\mathcal{T}_{NL}=\left\{t\left(\underbrace{0,0,...,0}_{(k)},\underset{(1)_{ij}}{D}{}^m,\underset{(2)_{ij}}{D}{}^m,...,\underset{(k)_{ij}}{D}{}^m\right)\in\mathcal{T}_N,(k\geq 2,k\in\mathbb{N})\right\},$$

$$\mathcal{T}_{N\underset{(1)}{C}}=\left\{t\left(\underbrace{0,0,...,0}_{(k)},B^m_{ij},0,\underset{(2)_{ij}}{D}{}^m,...,\underset{(k)_{ij}}{D}{}^m\right)\in\mathcal{T}_N,(k\geq 2,k\in\mathbb{N})\right\},$$

$$\mathcal{T}_{N\underset{(2)}{C}}=\left\{t\left(\underbrace{0,0,...,0}_{(k)},B^m_{ij},\underset{(1)_{ij}}{D}{}^m,0,\underset{(3)_{ij}}{D}{}^m,...,\underset{(k)_{ij}}{D}{}^m\right)\in\mathcal{T}_N,(k\geq 2,k\in\mathbb{N})\right\},$$

....

$$\mathcal{T}_{N\underset{(k)}{C}}=\left\{t\left(\underbrace{0,0,...,0}_{(k)},B^m_{ij},\underset{(1)_{ij}}{D}{}^m,\underset{(2)_{ij}}{D}{}^m,...,\underset{(k-1)_{ij}}{D}{}^m,0\right)\in\mathcal{T}_N,(k\geq 2,k\in\mathbb{N})\right\},$$

$$\mathcal{T}_{N\underset{(1)}{C}\underset{(2)}{C}}=\left\{t\left(\underbrace{0,0,...,0}_{(k)},B^m_{ij},\underbrace{0,0,...,0}_{(k)}\right)\in\mathcal{T}_N,(k\geq 2,k\in\mathbb{N})\right\}.$$

Proposition 2. $\mathcal{T}_{NL},\mathcal{T}_{N\underset{(1)}{C}},\mathcal{T}_{N\underset{(2)}{C}},...,\mathcal{T}_{N\underset{(k)}{C}}$ and $\mathcal{T}_{N\underset{(1)}{C}\underset{(2)}{C}...\underset{(k)}{C}}$ are Abelian subgroups of $\mathcal{T}_N$.

Proposition 3. The group $\mathcal{T}_N$ preserves the nonlinear connection N,$\mathcal{T}_{NL}$ preserves the nonlinear connection N and the component L of the local coefficients D$\Gamma(N)$, $\mathcal{T}_{N\underset{(1)}{C}}$ preserves the nonlinear connection N and the component $\underset{(1)}{C}$ of local coefficients $D\Gamma(N), ..., \mathcal{T}_{N \underset{(k)}{C}}$ preserves the nonlinear connection N and the component $\underset{(k)}{C}$ of the local coefficients $D\Gamma(N)$ and $\mathcal{T}_{N \underset{(1)}{C} \underset{(2)}{C} ... \underset{(k)}{C}}$ preserves the nonlinear connection N and the components $\underset{(1)}{C} \underset{(2)}{C}, ..., \underset{(k)}{C}$ of the local coefficients $D\Gamma(N)$.

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